UNIVERSIDADE DE LISBOA

INSTITUTO SUPERIOR DE ECONOMIA E GESTÃO

Agent-Based Models in Macroeconomics

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Orientadores: Prof. Doutora Tanya Vianna de Araújo

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RESUMO

A crise financeira de 2007/2008 desencadeou uma onda de críticas à teoria económica. Ataques baseados em quatro críticas principais: os economistas não terem previsto a maior crise desde a Grande Depressão; as autoridades deixarem formar bolhas sem controlo; o falhanço da supervisão bancária; e os modelos usados serem desfasados da realidade. No caso dos modelos, o alvo principal são os modelos DSGE (dinâmicos, estocásticos e de equilíbrio geral) e duas das suas hipóteses simplificadoras: o agente representativo e a racionalidade. As economias são realidades complexas, não-lineares e heterogéneas, e o recurso a métodos computacionais pode ser uma alternativa para ultrapassar as limitações dos modelos tradicionais. O objectivo desta tese é alargar a aplicação dos modelos de agentes em Macroeconomia com três exemplos distintos. O primeiro é um modelo de crescimento endógeno, de gerações sobrepostas, em que a decisão dos agentes sobre estudar é baseada na satisfação e na influência dos seus pares. É usado para testar os efeitos de longo prazo do paradoxo de Easterlin, que sugere que a satisfação e o rendimento não têm uma relação linear. Verifica-se que, no cenário de Easterlin, o crescimento é menor do que no cenário base onde os agentes atribuem igual importância ao rendimento absoluto e relativo. O segundo modelo visa avaliar o contágio dos *defaults* da dívida pública e a forma como as estratégias dos governos afetam o seu aparecimento e propagação. As simulações mostram que os países mais gastadores e com menor aversão ao risco tendem a entrar mais vezes em *default* e que políticas monetárias muito expansionistas podem originar fenómenos de risco moral. No terceiro modelo, estudamos o fenómeno da 'fuga de cérebros' e as consequências no crescimento económico. Concluímos que o efeito positivo do *brain drain* na acumulação de capital humano depende fortemente da probabilidade de emigrar.

Palavras-chave: modelos de agentes; crescimento económico; capital humano; default soberano; brain drain

ABSTRACT

The 2007/2008 financial crisis triggered a wave of criticism of the economic theory. These attacks are based in four main critics: economists had not foreseen the biggest crisis since the Great Depression; authorities let bubbles form without control; weak banking supervision; and the models used in macroeconomic policy being out of touch with reality. In the particular case of the macroeconomic models, the target are the DSGE models (dynamic, stochastic and general equilibrium) and their two simplifying hypotheses: the representative agent and rationality. Economies are complex realities, with nonlinearities and heterogeneities, and computational economics can be an advantageous alternative to overcome the shortcomings of the traditional models. The aim of this thesis is to extend the application of agent-based models to macroeconomic topics in three distinct models. The first one is an endogenous growth model, in an overlapping generations environment, in which the agents' individual decision to study is based on the satisfaction of their peers. It is used to evaluate the long-term effects of the Easterlin paradox, which states that satisfaction and income have a non-linear relation. The second model is used to study sovereign default contagion in order to assess how different government strategies affect default and propagation across countries. Simulations showed that high spending and low risk aversion levels are associated to a high prevalence of default and that monetary stimulus can create moral hazard problems. In the third model, we study the brain drain phenomenon and its economic growth effects. We conclude that beneficial brain hypothesis depend heavily on emigration probability.

Keywords: agent-based modelling; economic growth; human capital; sovereign default; brain drain

RESUMO ALARGADO

Nos últimos dez anos, depois da crise financeira de 2007/2008, a teoria económica e os seus modelos estiveram sob ataque cerrado. De dentro e de fora da profissão. A principal crítica aos economistas veio do facto de não terem previsto a chegada da maior crise desde a Grande Depressão. Mas não foi a única. Houve críticas às autoridades por terem permitido a formação de enormes bolhas em vários mercados e por não terem tido uma supervisão bancária suficientemente eficaz para evitar o colapso dos bancos. Uma crítica que teve os bancos centrais como principal alvo. Houve também um ataque feroz aos modelos usados na política económica, nomeadamente os modelos DSGE (dinâmicos, estocásticos e de equilíbrio geral) por falta de aderência à realidade.

Neste caso particular, a crítica aos modelos DSGE teve a ver essencialmente com duas das suas hipóteses simplificadoras: o agente representativo e a racionalidade. O agente representativo obriga a assumir um mundo excessivamente homogéneo quando a economia é uma realidade complexa cheia de heterogeneidade e comportamentos não-lineares. O que, aliás, está bem identificado pelas ciências da complexidade. Já a hipótese da racionalidade tem sido frequentemente disputada pela economia comportamental, com raízes na psicologia, e pela neuroeconomia, nascida das neurociências. Há, de resto, cada vez mais evidências de que a hipótese de racionalidade – ou as expectativas racionais – é irrealista face às reais capacidades cognitivas do ser humano. Estas duas hipóteses simplificadoras, usadas nos modelos DSGE mesmo em versões com influência keynesiana da Nova Síntese Neoclássica, são uma forma de permitir a sua solução analítica mas são uma limitação importante na sua aproximação à realidade.

Como realidade complexa, o estudo da economia pode beneficiar dos métodos da economia computacional, em particular dos modelos de agentes que são uma das mais conhecidas e visíveis instrumentos da economia computacional. A computação é hoje uma ferramenta fundamental em qualquer ciência. Os modelos de agentes permitem criar sociedades artificiais e economias de agentes heterogéneos que interagem entre si. Estão habitualmente associados ao estudo dos sistemas complexos, com aplicações que vão da Física, das Neurociências ou da

Biologia até à Economia ou Ciência Política. A vantagem deste tipo de modelos de agentes é que permitem analisar sistemas onde o seu comportamento estrutural e global não pode facilmente ser compreendido a partir da dinâmica dos agentes individuais. A partir de simulações, é possível identificar a emergência de padrões ou de comportamentos globais que, à primeira vista, não eram visíveis.

Ao contrário dos modelos tradicionais, que estão limitados por hipóteses simplificadoras, os modelos de agentes podem trabalhar com realidades complexas e heterogéneas. Teoricamente, com exceção da capacidade de computação *per se*, não há limites à heterogeneidade, à aleatoriedade e racionalidade que se pode incluir num modelo de agentes. O que é uma enorme vantagem para lidar com um mundo complexo e que permite ultrapassar claramente as limitações dos modelos que necessitam de resolução analítica e que, por isso, têm naturalmente que assentar em versões simplificadas da realidade.

A utilização de modelos de agentes não é nova em Economia mas a grande maioria dos exemplos abordam tópicos microeconómicos, relacionados, por exemplo, com os mercados financeiros ou decisões de consumo. Embora existam já aplicações a questões macroeconómicas, os exemplos são ainda minoritários. A ascensão dos modelos de agentes nos últimos nota-se na quantidade de artigos publicados mas também no facto de artigos com base nesta metodologia começarem a aparecer em revistas da especialidade onde, até há pouco tempo, o seu acesso parecia vedado.

O objetivo desta tese é alargar a aplicação dos modelos de agentes a temas macroeconómicos com três aplicações diferentes em termos de tema e de características dos modelos utilizados. É uma forma de demonstrar a utilidade e a diversidade de uma metodologia com resultados em muitas áreas da ciência e cuja aplicação à macroeconomia promete resultados promissores.

O primeiro modelo é um modelo de crescimento endógeno, com gerações sobrepostas, em que os agentes decidem sobre estudar a partir da influência e da satisfação dos seus pares. O objectivo é testar o paradoxo de Easterlin, que defende que não existe uma relação linear entre felicidade e rendimento, e a forma como este pode afetar o desempenho de longo prazo da economia. Mais concretamente, usamos uma função de satisfação em que o rendimento é a variável crucial mas com duas componentes distintas: a variação absoluta do rendimento (a forma como evolui no tempo) e a variação relativa (face aos seus pares). Nos cenários simulados, testámos situações extremas em que apenas o rendimento absoluto é relevante ou, pelo contrário, em que apenas o rendimento relativo pesa na satisfação. Verificou-se que o melhor desempenho económico acontece quando a satisfação individual depende, em iguais proporções, das componentes absoluta e relativa do rendimento. Quando a satisfação individual depende apenas da componente relativa, um cenário mais próximo do paradoxo de Easterlin, os resultados em termos de crescimento económico de longo prazo são piores. Já quando apenas a componente absoluta é considerada, a performance é muito semelhante ao cenário com pesos iguais.

No segundo modelo, os agentes são países que tomam decisões sobre endividamento num contexto de incerteza quanto ao rendimento e onde o nível geral de taxas de juro é determinado por um banco central. O objectivo é testar o fenómeno do *default* das dívidas soberanas e a forma como se propaga entre países, num contexto de heterogeneidade entre países e onde os governos têm diferentes perspetivas sobre o recurso ao endividamento. As principais conclusões das simulações foram: os países com maior nível de aversão ao risco (isto é, governos mais cautelosos no endividamento) têm menor prevalência de casos de *default*; os países 'gastadores' entram mais vezes em incumprimento; e, entre outras coisas, a política agressiva do banco central com juros demasiado baixos pode criar fenómenos de risco moral. Embora seja um modelo teórico, que até é uma adaptação de um modelo de teoria dos jogos, pretende dar algumas pistas sobre uma realidade próxima da que se viveu recentemente na zona euro na chamada crise da dívida soberana.

O terceiro modelo é igualmente um modelo de gerações sobrepostas através do qual se pretende avaliar o fenómeno do *brain drain*, mais concretamente a hipótese deste ser benéfico para o país de origem. Esta possibilidade tem a ver com a acumulação de capital humano e com o facto de, alguns dos agentes que decidiram investir em educação, não conseguirem emigrar porque apenas uma fração dos potenciais emigrantes consegue os seus intentos. Conclui-se que a probabilidade de emigração em cada período é uma variável crucial para determinar os efeitos económicos do *brain drain* e a sua relação com o crescimento do produto tem uma natureza não linear. Quando o nível de aversão ao risco dos agentes é maior, ou seja, o prémio que exigem para emigrar é superior, o desempenho de longo prazo da economia é menor.

Os três modelos de agentes apresentados pretendem dar uma visão panorâmica da utilidade deste tipo de instrumento na macroeconomia. Em termos de versatilidade e diversidade. Todos eles têm inúmeros caminhos por onde podem ser explorados e melhorados no futuro. Não apenas para se tornarem, em termos teóricos, mais próximos da realidade que pretendem retratar, mas também em termos utilização de dados reais e até, eventualmente, da sua aplicação a outros tipos de fenómenos com comportamento semelhante. Os modelos de agentes em macroeconomia ainda estão a dar os primeiros passos. O caminho é muito promissor mas é ainda bastante longo.

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1. INTRODUCTION

The financial crisis has triggered a wave of criticism about economic theory, its models and the way Economics is taught in the universities. A wave that spread worldwide with some anecdotal events such as Harvard students' refusal to attend a Gregory Mankiw class on Principles of Economics in 2011 It was also during this period that, by the hand (and money) of George Soros, the Institute for New Economic Thinking was born with the aim of "challenge conventional wisdom and advance ideas to better serve society".

The crisis was also a time of glory for Hyman Minsky, the american economist who studied at the University of Chicago and whose analysis of financial crises and tipping points were finally properly acknowledged two decades after being published. There were several Minsky moments without most of us knowing that they had been already baptized with that name. Paul Krugman, himself a crisis scholar and a Nobel laureate in Economics in 2008, only in the midst of the subprime burst have the first real contact with Minsky ideas[1].

Over the past ten years, Economics and its theories and models have been under attack. From outside, but also from within Economics, in the campuses and the profession. There is a fierce criticism against (some) economists for not foreseen the biggest crisis since the Great Depression, against authorities (namely central banks) for letting huge bubbles to form, against inefficient bank supervision or against theoretical models used to analyze the economy for being out of touch with reality.

On this last point, the big target are the DSGE models (dynamic, stochastic and general equilibrium) and, in particular, the fact that they rely on two simplistic assumptions: agents are represented by a representative agent (homogeneous in a world of heterogeneity) and are assumed to be rational. These limitations are well identified by the Sciences of Complexity, that deals with heterogeneity and non-linearity, and behavioral and neuroeconomics that presents very strong evidence about cognitive and decision-making capabilities of the agents in contrast with rationality hypothesis.

[1] Conscience of a Liberal, The New York Times blog, may 19th 2009

There was even an article by Paul Romer[2], the 2018 Nobel laureate in Economics, criticizing the fascination with mathematics of the models and papers where excessive mathematical formalism, rather than being a tool to understand reality, became an end by itself.

In this context, computational economics and, in particular, agent-based models gained a huge pertinence in economic analysis. And increased its presence in Economics journals in terms of the number of published papers. But it is still far away far from being a mainstream methodology, tough it is now referred and considered by more and more economists – some of them important 'names' in the traditional paradigms – and it is increasingly being used in different topics and appears in journals where, until recently, they had virtually no access at all.

The purpose of this thesis is to broaden the application of agent-based models to macroeconomic issues. Microeconomic topics are still, by large, the most frequent applications of agent-based models in Economics. But there are, of course, several examples of agent modeling applications to macro issues and even intersections with DSGE models, as we will see in chapter 2.

Chapters 3, 4 and 5 present three different applications of agent-based models to macroeconomics. The first one is an overlapping generation endogenous growth model where individual decisions about studying are based on the agents´ satisfaction and on their peer (neighbors) influence. This model is used to test the Easterlin paradox (satisfaction and happiness are not completely linear with relation to income), namely the relationship between income and satisfaction and its consequences for long-term growth. The second model, in chapter 4, focus on countries' sovereign debt default contagion phenomena in an adaptation of Jean model for game theory. Our aim is to assess how more 'spending' or 'cautious' strategies of the governments are related with sovereign default and how these crises spread across countries. In the third article (chapter 5), we study the brain drain phenomena – a topic much debated in Europe and Portugal during the 'austerity' years – and, in particular, the beneficial brain drain hypothesis, which states that brain drain can have positive effects in source economy in terms of human capital accumulation. We provide the full

[2] Romer, P. M. (2015). Mathiness in the theory of economic growth. *American Economic Review*, *105*(5), 89-93.

description of the models according to ODD (Overview, Design, Details) protocol at the end of each chapter.

In Chapter 2, we present some literature review, with the evolution of agent-based models, their applications in Economics and macroeconomics, how can they address the shortcomings of traditional macroeconomic models and some basics of the construction of an agent-based model in Macroeconomics. Chapter 6 concludes.

2. AGENT BASED MODELS IN MACROECONOMICS

Agent-based models are one of the most salient and well-known methods used in computational economics, although there are several others with significant development in recent years. Computational power is today a very important tool in every science and Economics is not an exception. These agent-based models can create artificial societies and economies of interacting heterogeneous agents and are normally associated with the study of complexity and complex systems, with applications ranging from Physics, Neurosciences and Biology to Economics or Political Science. Because, as Araújo (2011, p.19) underlines, the structural behavior of these systems, especially when they have a very large number of agents, "cannot be easily predicted from the dynamics of the individual agents". Agents can be varied "as molecules, cells, living organisms, animal groups, human societies, industrial firms or competing technologies" and present three fundamental properties: emergency, non-linearity and interdependence.[3]

According to Gilbert (2008, p.2), agent-based modelling is "a form of computational social science", which "involves building models that are computer programs" and "creates some kind of simplified representation of ''social reality''. The complexity of these systems requires tools to deal with their properties and to detect local (individual-level) and global dynamics (system-wide). This approach is obviously useful in many areas and, in particular in Economics, where agent-based models can mix micro and macroeconomic dimensions with a degree of freedom (and assumptions) that traditional models do not permit.

Different complex systems in nature, "such as insect colonies, immune systems, brains, and economies, have much in common", says Mitchel (2009, p.4). For that reason, Economics should not turn its back to these kind of tools and technical solutions capable of overcoming the limitations of traditional models. In fact, economists are already using agent-based models to explain markets behavior and, among other examples, to detect global patterns like bubbles, crashes or sudden stop phenomena (Mitchel, 2009, p.10).

[3] For a more detailed definition of agent models and how they fit into the broader area of computational economics see Araújo (2011).

Tesfatsion (2006, p.833) have no doubt that “economies are complex dynamic systems” with “large numbers of micro agents engage repeatedly in local interactions, giving rise to global regularities” and the usefulness of agent-based models and other complexity tools, computational or not, in Economics is undisputed.

Complex systems have a perfectly typified set of common characteristics. Mitchel (2009, pp. 12-14) lists three: complex collective behavior (simple rules without a leader, collective actions that result from various components and patterns of behavior that are changing); signal and information processing (external and internal signals); and adaptation (through evolutionary or learning processes). Tesfatsion (2006, pp. 836-837) takes the two properties – system composed of interacting units and exhibiting *emergent* properties arising from the interactions – listed by Flake (2008), recognizes the difficulty in reaching a consensual definition of complex adaptive system and leaves us with three definitions to cover the different possibilities: a system that includes reactive units with different attributes in reaction to changes in environment; a system that includes goal directed units whose reactions are based on the achievement of objectives; a system that includes planner units that are goal directed and exert some control of the environment to achieve their goals.

Agent-based models did not 'born' in Economics but have been increasingly used in recent decades, both in terms of the number of publications and the range of topics covered. Are “the best known part of computational economics” and it is its component “that comes closest to a laboratory approach; where each execution of the model simulates the behavior of a society that, by existing in a computational environment, is called artificial society or artificial economy” (Araújo, 2011, p.221). There are four main objectives in agent-based model research (Tesfatsion, 2006, pp.838-842): empirical understanding (from real data); normative understanding (using models to improve policy design); theory generation (attempt to better understand the functioning of the economy from its dynamics); and methodological advances (improve the study of economics).

Traditionally, in its first applications, agent-based models were mainly concerned with microeconomics. But, as we will see later in this chapter 2, they have been (and can be) used to address many different macroeconomics topics, such as endogenous growth (examples in Araújo

2011, pp.226-228), economic cycles, or monetary policy. We will look more closely to these applications in macroeconomics in section 2.2.

Hommes (2013, p.1) explains that computational methods and agent-based models fits perfectly well to Economics because the economies are complex systems with nonlinear properties and feedback loops. This view, he adds, traces back to Joseph Schumpeter, Friedrich Hayek, and Herbert Simon. Since the 1980s, the complexity model paradigm "been strongly advocated since the 1980s by economists and multidisciplinary scientists from various fields, such as physics, computer science and biology, linked to Santa Fé Institute". More recently, it is being used in economics also by policy makers trying to understand some economic phenomena that normally is not very well captured by traditional models.

Computational economics and agent-based models are a very promising methodology to hypothesis testing or scenario comparison that cannot be done with the traditional macroeconomic tools. That does not mean, however, that mathematical analysis should be abandoned or being relegated to a simple complementary role because it is different from agent-based models in four dimensions (De Marchi and Page, 2014, pp. 11-12): first, in agent-based models analytical tractability is not a main concern as happens with deductive models; second, game-theoretic and agent-based models are different, but not that different because concepts such as equilibrium, symmetry or efficiency are present in both methods and, frequently, they can be used in a complementary manner; third, both models can use preferences and utility functions but, unlike deductive models, these can normally evolve in agent-based models; fourth, though both types of models can address the same topic, they are usually used in different situations (for example, using deductive models for equilibrium purposes and agent-based models for phenomena like crashes or sudden stops.

"Whatever name is used, the purpose of agent-based modeling is to understand properties of complex social systems through the analysis of simulations", says Axelrod (1997, p.3). This method, he explains, contrasts with the standard induction (using data to find patterns) and deduction (using axioms and mathematics to derive consequences). Axelrod recognizes that "there are some models, however, in which emergent properties can be formally deduced" – for example, the neoclassical model

– but whenever the agents have adaptive instead of optimizing strategy is impossible to use deductive methods. It that cases, simulation is necessary.

To Gilbert and Terna (2000, p.3), “statistical and mathematical models also have some disadvantages” and the main one is related with the fact that the equations necessary to represent the reality are too complex to be analytically tractable. Which is particularly notorious, says this author, when the object of the analysis involve non-linear relationships because, in that cases, the solution is often to oversimplify the equations. The model will become solvable at expenses of the adherence to reality.

Farmer and Foley (2009) have no doubts about the utility of agent-based models compared with other available alternatives, because these models do not rely on “a predetermined equilibrium state” and, at any moment, the agent´s actions are based on the current environment and the rules governing their behavior. Why aren´t these models more widely used? They explain that it is the consequence of “historical choices made to address the complexity of the economy and the importance of human reasoning and adaptability.” A reason that dates back to the Keynesianism ‘failure’ to explain stagflation and to rational expectations revolution that followed.

Delli Gatti, Gaffeo and Gallegati (2010, p.8) are very critical about the traditional models – “one is allowed to use a hierarchical reductionist approach if and only if the interaction between elementary units is linear” – and say that, if we take seriously the Lucas´ critique[4], we should realize that the standard micro foundation methodology must be discarded “simply because it is incorrect”.

Leombruni and Richiardi (2005) analyzed precisely why economists are sceptical about agent-based models and concluded that the skepticism is related with the difficulties in interpretation and generalization of the results but also with the estimation of the model itself, namely the fact that a richer model could raise under identification problems.

The first signs of ‘chaos’ in Economics are older than one might think. Economists like John Hicks, the father of IS-LM model, used implicitly complexity and non-linear analysis even though, at the time, they don´t have the computational firepower available today (Hommes, 2013, p.3) [5].

[4] Lucas (1976)

[5] For a more detailed description about chaos theory see, for example, Gleick (2011).

One of the earliest social science models is the William Phillips (1950) hydraulic model of the economy with water flowing through pipes and vessels to represent the circulation of money (Gilbert 2008, p.4)[6].

It should be stressed, however, as De Marchi and Page (2014, p.2) do, that agent-based models and complex systems are not the same thing. Agent-based models are a tool and complexity refers to systems and processes hard to explain, predict or forecast. Nevertheless, agent-based models area one of the most powerful instrument to understand complex systems, in particular because of their potential to analyze unexpected events and tipping points with better results than traditional models (Lamberson and Page, 2012).

Thomas Schelling (Schelling, 1971) segregation model is one of the first agent-based models known. It was used to study household localization – the agents – and how they are racially divided. According to Gilbert (2008, p.7), it became very influent for four reasons: the outcome is surprising and is was not predictable just looking to the individual agent decision rule; it is very simple with only one parameter (tolerance threshold); the emergent clustering behavior; the possibility of testing the model with empirical data. Other classic examples are the studies about vote and opinion dynamics, industrial networks or consumer behavior. [7] All these are references today and paved the way to several different applications of agent-based models in many areas.

Agent-based models are simply a method to assess how individual agent behaviors aggregate and how different assumptions about their characteristics, decisions or interdependencies, affects the economic variables, such as economic growth, unemployment or financial markets. The great advantage is that they allow us to artificially simulate macro level from the individual level and thus draw conclusions that would be difficult to achieve in traditional models. As stated by Araújo (2011, p.224), in economic systems, the creation of structures by bottom-up processes is analyzed when we investigate the reason behind the emergence of global

[6] This model can be admired at the Science Museum, London. Water flows are used to simulate variations in parameters of the model, such as interest rates.

[7] For a detailed description of some of the most well know models and its applications in Economics, see for example Gilbert (2008, pp.6-14).

characteristics even when the economies don´t have a central planner or control measures traditionally associated with those characteristics.

Agent-based models, as we will see in the following section, are an instrument to incorporate heterogeneity in the models and to consider agent´s characteristics that deviates them from the traditional assumptions of macroeconomic models in terms of rationality. In recent decades, the rationality of *Homo Oeconomicus* have been challenged by behavioral economics and the neuroeconomics.

In fact, there are already many behavioral economics applications, even in macroeconomics. "Behavioral economics is an umbrella of approaches that seek to extend the standard economics framework to account for relevant features of human behavior that are absent in the standard economics framework" (Diamond and Vartiainem, 2012), which means using social sciences toolbox, namely psychology or sociology. One of the most prosperous and frequent topic is finance but there are many other examples[8].

Fagiolo and Roventini (2012, pp 16-17) present the ten ingredients of agent-based models in Economics: bottom-up perspective (aggregate properties and patterns obtained from individual behavior); heterogeneity (heterogeneous agents); evolving complex system (agents live in a systems that evolves through time); non-linearity (non-linear interactions and feedback loops); direct endogenous interactions (agent interact with each other directly); bounded rationality (systems too complex to have hype-rational agents) ; nature of learning (agents adapt to changing systems); 'true' dynamics (system evolves in path-dependent manner); endogenous and persistent novelty (economic systems non-stationary); selection based mechanisms (agents face some kind of selection mechanism).

Windrum, Fagiolo and Moneta (2007, p.2) have a more synthetic version with just three ingredients: bottom-up perspective; bounded-rational agents; and networked direct interactions. And they take three consequences from here: agents learn by engaging in an open-ended search of dynamically changing environments; agent-based models have a non-

[8] Diamond and Vartiainem (2012) presents six areas where behavioral economics are useful: public economics, development, law and economics, health, wage determination and organization economics.

reversible dynamics and the system evolves in a path-dependent manner; sometimes the models have selection-based market mechanisms.

2.1 Traditional Models versus Agent-Based models

The financial crisis and the apparent surprise it caused in many economists, supervisors and government officials, raised huge doubts about the reliability of the macroeconomic models used in policy making. The main target were dynamic stochastic general equilibrium (DSGE) models – or other similar models – which had a very important role in central banks and governments toolkit. In July 2009, the British magazine "The Economist" ran a cover story 'declaring' the death of Modern Economic Theory, criticizing precisely DSGE models. Paul Krugman (2009 and 2012), in The New York Times Magazine but also in an academic paper, also gave voice to some of the public outrage against the economists and their models.

In 2009, the Hungarian billionaire and philanthropist, George Soros, created the London-based Institute for New Economic Thinking claiming that "we are economists who challenge conventional wisdom and advance ideas to better serve society". More precisely, new economic thinking includes six key principles: economists and their ideas must be independent from powerful interests; complexity and uncertainty are inherent in economic and financial systems; inequality and distribution matter as much as growth and productivity; heterodox models that pose alternatives to the neoclassical orthodoxy are essential; history matters; diversity of race, gender, class and other forms of identity enrich economic thought; an outdated economic structure is endangering our planet but new approaches can save it; multidisciplinary learning.

One of the names that have a 'second life' in the financial crisis aftermath was Hyman Minsky and his financial instability hypothesis (Minsky 1986 and 1992). Minsky wrote it many years before the subprime burst in US but the idea regained attention around the world after 2007/2008.

There are real problems with the macroeconomic models used, such as the lack of financial systems, the impossibility of financial crises or the assumptions about economic agent´s rationality and homogeneity. An oversimplification related with the need of having model suitable for mathematical analysis. Delli Gatti, Desiderio, Gaffeo, Cirillo and Gallegati summarizes de critics (2011, pp.1-2): “Contemporary economics is in troubled waters. This is true most of all for that particular area of the economic discourse labeled macroeconomics. Although in our days there exists a consolidated and celebrated mainstream framework known as Dynamic Stochastic General Equilibrium (DSGE) model (Blanchard, 2008; Woodford, 2008), its internal coherence and ability in explaining the empirical evidence are increasingly questioned from several quarters (Colander, 2006; Howitt *et al.*, 2008; Juselius and Franchi, 2007), especially after the turmoil of the first global crises of the 21st century has materialized almost unannounced and misconstrued (Driffill, 2008).”

The root of the problem, they explained, dates back to the 18th century, one century after the Newtonian revolution, when the mechanical physics of the 17th century inspired economists like Stanley Jevons, Carl Menger and Leon Walras and the marginalist revolution in which the selfish, rational and utility maximizing economic agent (*Homo Oeconomicus)* is the centerpiece. This approach to human behavior was “firmly rooted on the holy trinity of classical physics, i.e. reductionism, determinism and mechanicism”, precisely when other domains of knowledge were questioning the universality of Newton mechanical model as universal.

Delli Gatti, Desiderio, Gaffeo, Cirillo and Gallegati (2011, p.7) emphasize that, in recent years, macroeconomics converged to a “commonly accepted paradigm”, the *new neoclassical synthesis* (NNS) (Goodfriend and King, 1997), whose most visible and used model are precisely DSGE models: “The main idea behind the NNS rests on the blending of key elements of neoclassical real business cycle theory with key elements of the new Keynesian tradition of the 1980s.” We will see in detail what this means in the section 2.1.2.

2.1.1 Traditional models hypothesis under criticism

Many macroeconomic models, notably DGSE models, work with a representative agent and assume, moreover, that the agent is rational, a true *Homo Oeconomicus*. Two fundamental hypotheses to allow the mathematical treatment of these models, but with very little adherence to economic reality.

Using agent-based models it is possible to overcome these two limitations by introducing numerous forms of multidimensional heterogeneity. It is possible to have individualized agents, as happens, for example, in the model of satisfaction of chapter 3.1 or in the brain drain model with the ability to learn individually in chapter 3.3.

The basic assumptions of DSGE models – representative agents, rational expectations, among others – "prevent the understanding of basic phenomena underlying the current economic crisis", refers Fagiolo and Roventini (2012). And they underline the fact that instead of performing "Ptolemaic exercises (Stiglitz, 2011; Dosi, 2011; Caballero, 2010)" trying to fix DSGE models with more and more frictions (the new-Keynesian approach), economists should instead consider the economy as a complex system: "This is the starting point of agent-based computational economics (Tesfatsion, 2006; LeBaron and Tesfatsion, 2008). Bounded rationality, endogenous out-of-equilibrium dynamics, direct interactions, are the tenets of agent-based computational economics which allow to catch many of the features of the current crisis."

Agent rationality is disputed by behavioral economics, rooted in psychology, and the neuroeconomics, which comes from neurosciences to challenge the decision-making ability that many macroeconomic models assume. Sunstein (2013, p. 12-21) presents four behavioral market failures based on the systems 1 and 2 (slow and fast) defined by Kahneman (2011): present bias and time inconsistency (in standard models, agents consider simultaneous short term and long term but, in practice, people frequently show bias discounting future or procrastinating); ignoring important attributes (attention is a scarce resource and sometimes people miss important aspects)[9]; unrealistic optimism (predictions and prospects skewed in optimistic direction); probability problems (agents do not deal with probability in a proper way and frequently over or underestimates de risks involved).

Mullainathan and Thaler (2000, p.3) consider that "the standard economic model of human behavior includes (at least) three unrealistic traits: unbounded rationality, unbounded willpower and unbounded selfishness". Regarding unbounded rationality they mention the seminal work on this

[9] Reis (2006 and 2006a), Mankiw and Reis (2002) and Ball, Mankiw, and Reis (2005) presents some examples of neokeynesian models with inattentiveness features.

topic by Herbert Simon (1955), but also Kanheman and Tversky heuristic consequences and prospect theory (1974 and 1979, respectively) and Camerer loss and mental accounting (1997), among others. About willpower, they remember that frequently "people have self-control problems" and even when they have perfect knowledge about the 'right' choices they might make the wrong choice. The same happens with unbounded selfishness.

Hommes (2013, pp. 5-10) underlines the fact that "the most important difference between economics and the natural sciences is perhaps the fact that decisions of economic agents today depend upon their *expectations* or *beliefs* about the future". The expectations have a very important role in decisions and, depending on its formalization, the outcome of the models can be very different. Besides, agents have bounded rationality, which means that they don´t have all the capacities that some models give them related with information gathering and computing. That is why some economic models already assumed adaptive expectations.

At the same time, Hommes says, "the representative agent model has played a dominant role in modern economics for quite some time. Most rational expectations models assume a single, *representative agent*, representing average consumer, average firm or average investment behavior." The idea of rational agents "dates back to the 1950s, to Milton Friedman (1953) who argued that non-rational agents will be driven out of the market by rational agents, who will trade against them and earn higher profits". "In recent years, however, this view has been challenged and heterogeneous agent models are becoming increasingly popular in finance and in macroeconomics (Kirman 1992, 2010)". And agent based models are an important method to deal with all this heterogeneity[10].

Bernheim and Rangel (2007, pp. 10-20), for example, discuss the typical agent´s preferences assumptions – coherence, domain, fixed lifetime and no mistakes – and the consequences of relaxing them based on behavioral and neuroeconomics inputs in terms of welfare and public policies. And Bernheim (2008) presents the impact of neuroeconomics in "positive

[10] For some examples of theoretical models using this heterogeneity see Hommes (2013, pp. 10-36).

analysis of decision making" (pp. 3-42) and on normative economics (pp. 42-54)[11].

2.1.2 DSGE models

Now, we will analyze DSGE and its hypothesis in detail. Tracing back its path from the beginning and its origins until the so called New Neoclassical Synthesis (NNS)[12]. NNS emerged from the confrontation of two alternative research streams that converged and became unique: Real Business Cycles (RCB) e new-Keynesian paradigm that introduced frictions in the models. As Fagiolo e Roventini (2012. p.4) put it: "In a nutshell, the canonical model employed by the NNS paradigm is basically a RBC dynamic stochastic general equilibrium (DSGE) model with monopolistic competition, nominal imperfections and a monetary policy rule."

These models started with a stochastic version of the standard neoclassical growth model[13], with households represented by an utility maximizing representative agent with an infinite time horizon and many firms producing an a homogeneous good. All the agents involved are assumed to be rational. The Keynesian features of the model are related with: money, monopolistic competition and sticky prices. Unlike traditional RBC models, the economy can deviate from steady state for some time and monetary policy, who manages money supply, can act in the short term to put the economy back on track, i.e., to put output and unemployment again in their 'natural' level. Prices are sticky in Calvo terms (Calvo, 1983) and monopolistic competition is defined with a Dixit-Stiglitz model (Dixit and Stiglitz 1977).

NNS model is represented by three equations: (1) IS equation for aggregate demand; (2) new Keynesian Phillips curve for aggregate supply; and (3) a Taylor-type rule for monetary policy. Analytically, we have:

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[11] In neuroeconomics, however, no consensus have emerged yet. See, for example, the interesting debate between "Neuroeconomics: Why Economics Needs Brains" (Camerer, Loewentsein and Prezelec, 2004) e "The Case for Mindless Economics (Gul and Pesendorfer, 2008).

[12] Based in Fagiolo e Roventini (2012, pp.4-7). The authors says that New Neoclassical Synthesis it is not a absolute convergent definition. "This term was first introduced by Goodfriend and King (1997)", then Woodford (2003) labeled the approach as "Neo Wicksellian" and Gali and Gertler (2007) prefer the term "New Keynesian" the most used, even if earlier New Keynesian models were very different from the ones of the New Neoclassical Synthesis. For more detailed presentations of DSGE, NNS and neo-keynesian models and its applications «see, for example, Smets and Wouter (2007), Gali (2008) or Wickens (2012).
[13] Original developed by Solow (1956) and Swan (1956).

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t^n) \quad (1)$$

$$\pi_t = k\tilde{y}_t + \beta E_t\pi_{t+1} + u_t \quad (2)$$

$$i_t^\tau = r_t^n + \phi_\pi\pi_t + \phi_y\tilde{y}_t \quad (3)$$

where $\tilde{y}$ is the output gap; , i is the nominal interest rate; σ is the intertemporal elasticity of substitution of consumption, π is inflation, $r^n$is the 'natural' interest rate; β is the discount factor of the representative household; E_t is the expectation operator; k depends on the elasticity of marginal cost with respect to output and on the sensitivity of price adjustment to marginal cost fluctuations (i.e., frequency of price adjustment and real rigidities induced by price complementarities); u is a cost-push shock: it captures the fact that the natural level of output may not coincide with the socially efficient one for the presence of real imperfections in the markets; i_t^τ is the central bank interest rate target; $\phi y > 0$ and $\phi\pi > 1$.

As we can clearly see, IS equation has a typical behavior of the classic IS-LM model with a negative relation between interest rate and interest rate gap. Aggregate supply equation – the New Keynesian Phillips curve – depicts a relation between current inflation and inflations expectations, output gap and a cost shock that have an influence in inflation beyond two other factors. In the monetary policy rule, we have a standard formulation with nominal interest rate fixed by the central bank depending on the natural interest rate and two reaction parameters to inflation (which is above 1) and output gap (with a positive value).

Delli Gatti, Desiderio, Gaffeo, Cirillo and Gallegatti (2011, pp. 10-14) summarizes some of the "most relevant" inconsistencies of mainstream NNS: general equilibrium (GE) is neither unique nor locally stable (Sonnenschein 1972, Debreu 1974 and Mantel 1974); representative agent; general equilibrium computability (the GE solution is incomputable from a recursion theoretic perspective); price mechanisms (real markets work differently and operates in real time (Arrow, 1959)); money (agents cannot decide to monetize alone as models assume); absence of time .

The two most contested features of DSGE are the representative agent and the rational expectations hypothesis. The first one states that all the households are equal in a perfect homogeneous society. And the second assumes a cognitive capacity that is disputed by several studies in psychology and neurosciences. Today, these two assumptions, used for mathematical purposes, are even contested by some of the most vigorous defenders of DSGE models[14].

As De Grauwe (2010) underlines: "My contention is that macroeconomic models that use the rational expectations assumption are the intellectual heirs of these central planning models. Not in the sense that individuals in these rational expectations models aim at planning the whole, but in the sense that, as the central planner, they understand the whole picture. Individuals in these rational expectations models are assumed to know and understand the complex structure of the economy and the statistical distribution of all the shocks that will hit the economy." De Grauwe presents a behavioral macroeconomic model that contrasts with DSGE model: it has also three equations but with different features. In particular, the model introduces differences in expectation formation mechanisms for future output and inflation based on heuristic rules[15].

DSGE models have, of course, other features that were also fiercely questioned after financial crises. We can list three, at least: financial or a banking sector are frequently excluded from the models tough they are often the source of the financial crisis; models don't even admit the possibility of crisis and only accept supply random shocks, and there are many other oversimplifications beyond representative agents and rational expectations[16].

Gatti, Gaffeo and Gallegati (2010, pp. 6-8) consider that micro founded macroeconomics − exemplified by the DSGE models – "has been locked into

[14] VoxEU ebook "DSGE Models in the Conduct of Policy: Use as Intended" (eds. Gürkaynak and Tille, 2017) discusses the DGSE models role in policy making and have contributes, among others, from Olivier Blanchard and Jordi Gali about the future of this tool used by economists worldwide.

[15] De Grauwe used fundamentalists and extrapolative agents, regarding the rules they use to form expectations based on behavioral financial literature.

[16] For a more detailed discussion about empirical and theoretical validation of DSGE models in policy-making see (Fagiolo e Roventini, 2012, pp.7-16).

a wrong trajectory”. For three reasons: first, as recognized even by their proponents (Lucas and Sargent, 1981), the Lucas critique is theoretically empty; second, micro foundations of macroeconomics requires policy invariant microeconomic principles – preferences, for example − but, as experimental results shows (Bowles, 1998) that´s not really true and preferences are, in fact, endogenous; third, representative agents preferences mimics microeconomics individual behavior and perfect aggregation requires characteristics (homothetic preferences, for example) with low adherence to reality.

On the contrary, “agent-based models configure themselves as a very powerful device to address policy questions in more realistic, flexible and modular frameworks” (Fagiolo and Roventini, 2012, pp. 21-22). Agent-based models have two sort of advantages: theoretical (do not impose consistency requirements, nor demanding conditions to be analytically solvable) and empirical (allows to take data more easily and can explain many different pieces of evidence at the same time).

Hamill and Gilbert (2016, pp.238-239) underlined the fact that “agent-based models can handle heterogeneity, dynamics and interaction, and we have shown how it can be used to bridge the gap between micro and macro”. But they alert too to the existence of “real and major problem with the use of agent-based models in economics and that is the lack of standardization”.

Miller and Page (2009, pp. 78-89) remember that, when choosing between models, there is frequently a precision-flexibility trade-off and flexibility is precisely one of the reasons, among others (adaptive agents, inherently dynamic, heterogeneity or scalability, for example), that supports the use of agent-based models.

Farmer and Foley (2009) say that “the economy needs agent-based modelling” because “the cure for macroeconomic theory, however, may have been worse than the disease”. Rational expectations hypothesis “assumes that humans have perfect access to information and adapt instantly and rationally to new situations, maximizing their long-run personal advantage” and, of course, “real people often act on the basis of overconfidence, fear and peer pressure — topics that behavioral economics is now addressing”. But there is a larger problem “even if rational expectations are a reasonable model of human behavior, the mathematical machinery is cumbersome and requires drastic simplifications to get tractable results". That´s why "agent-based models potentially present a way to model the financial economy as a complex system, as Keynes attempted to do, while taking human adaptation and learning into account, as Lucas advocated".

For Windrum, Fagiolo and Moneta (2007, p.3), despite the "significant success" of agent-based modelers in recent years and the fact that neoclassical models already incorporate some features from complexity – heterogeneity or bounded rationality, just to mention two examples – "orthodox economists have not been moved to join the agent-based camp". And they have an explanation for this which is based in the fact that, while neoclassical economists "developed a core set of theoretical models and applied these to a range of research areas" while in the agent-based models side the "sheer diversity of alternatives (…) is striking".

In fact, empirical research and laboratory economics provides some results supporting agent-based models. Duffy (2014) provides a survey about laboratory research in macroeconomics that question precisely some of the traditional assumptions. Duffy (1998) surveyed the monetary theory laboratory experiments.

2.2 Examples of agent based models in macroeconomics

Most of the examples of applications of agent-based models in economics are focused in microeconomic and financial market topics, although in recent years more and more macroeconomic examples have emerged. In this section, we will briefly review some of the agent-based models in macroeconomics. There are examples in analyzing growth, economic cycles, innovation or economic policy (monetary and fiscal).

Biondo, Pluchinho and Rapisarda (2012) present an agent-based model, based in a Netlogo model, to analyze brain drain and, in particular, the decision to return home after the migration. In particular, their agents have two distinctive individual features which are important for the decision-taking: risk aversion and expectations.

Araújo and St. Aubyn (2008) use an agent-based model to test some different scenarios of endogenous growth and human capital accumulation. In this overlapping generation model, agents decide to study depending on their neighbors influence and on the expectations of future income and this individual dynamic develops macro-patterns in terms of GDP growth, among other results.

Riccetti, Russo and Gallegati (2015) present a macroeconomic agent-based model, with microeconomic features, which comprises four different markets (goods, labor, credit and deposit) and three types of agents (firms, households and banks), to evaluate questions like endogenous business cycles, Phillips curve or financial instability.

Dosi, Giovanni, Fagiolo, Napoletano, Mauro, Roventini and Andrea (2013) mixed Keynesian features, Schumpeterian innovation fueled process and Hyman Minsky credit dynamics in an agent-based model. With this kind of hybrid model, they conclude that fiscal policy is useful to manage business cycles fluctuations and that in monetary policy have a non-linear relation with macroeconomic dynamics.

Raberto, Teglio and Cincotti (2008) used an agent-based model to perform monetary experiments, i.e., to test interest rate setting rules based on output gap. Their model has one monopolistic firm, a central bank, a trade union and N different agents that are, simultaneously, consumers, workers and financial traders. Cincotti, Rabetto and Teglio (2010) tested credit and macroeconomic instability in an agent-based model and simulator Eurorace[17].

Gualdi, Tarzia, Zamponi and Bouchaud (2014) present a comprehensive detailed evolution in agent-based models in macroeconomics, from the model they called Mark 1[18] until many other published in recent years.

In their book "Macroeconomics from the bottom up", Delli Gatti, Desiderio, Gaffeo, Cirillo and Gallegati (2011, pp.45-8), present a detailed agent-base macroeconomic model with its main components, and application in areas such as credit market, consumption, R&D and growth, among others.

[17] Deissenberg, Van Der Hoog, and Dawid (2008).

[18] These model is based on different proposals that can be found at Gaffeo, Delli Gatti, Desiderio, Gallegati, (2008); Delli Gatti, Palestrini, Gaffeo, Giulioni, e Gallegati (2008); Delli Gatti, Desiderio, Gaffeo, Cirillo and Gallegati. (2010) and Russo (2013).

As Tesfatsion (2006, p.864) puts it, the use of agent-based economics could facilitate the development and experimental evaluation of integrated theories that build on theory and data from many different fields of social science. This means that, using these tools, it is possible to deal with topics such as economic growth, welfare, monetary and fiscal policy or redistribution issues. “Another potential important aspects is pedagogical”, the author continues, because it can be used in computational laboratories and “students can formulate experimental designs to investigate interesting propositions of their own devising, with immediate feedback and with no original programming required”.

In more general perspective, Mitchel (2009, p.301) considers that “complex systems science is branching off on two separate directions”. “Along one branch, ideas and tools from complexity research will be refined and applied in an increasingly wide variety of specific areas”, from physics to biology, sociology to political science and, of course, economics which is our object in this thesis. As we saw earlier, the frontiers of the utilization of complexity methods in economics – and agent-based models in particular – is expanding every year. Today, the areas and topics covered are wide but, nevertheless, is there room for further different applications. “The second branch, more controversial, is to view all these fields from a higher level, so as to pursue explanatory and predictive mathematical theories that make commonalities among complex systems more rigorous, and that can describe and predict emergent phenomena”.

Gualdi, Tarzia, Zamponi and Bouchaud (2014) have a detailed description of some possible macroeconomic utilization of agent based models, ranging from unemployment, monetary policy or wages and inflation.

2.3 How to build an ABM in macroeconomics

All the examples presented in section 2.2 are a useful departing point to anyone who wants to build an agent-based model in macroeconomics because they have enough diversity in terms of topics covered. But there are, of course, more general and theoretical aspects of agent-based model construction that should be stressed. In this section, we will present some of the main guidelines and features that should be presented in any agent-based model in macroeconomics.

According to Araújo (2011, pp.226-229), computational economics models can be divided into four groups: numerical models based on statistical analysis an numerical methods; computational models including models with artificial intelligence techniques, genetic algorithms or neural networks; economic models generally with microeconomic base but broadened to address macroeconomic questions; and agent-based models with predominant network approaches.

In terms of structure, Araújo (2011, p.229) considers that among the most frequently used concepts in agent modeling are: agents (autonomous element with its own dynamics and interdependent relations with other agents and the environment); environment (own entity or set of all agents); utility function (for accounting the agent performance); strategy (target-oriented options for each agent); learning (possibility for the agent to use information to improve his performance); memory (use of information from past iterations); iteration (fundamental component of model dynamics that acts as a timekeeping indicator); and simulation (model execution between start and end).

The dynamics of agent models, according to Araújo (2011, pp. 233-235), can be organized in six stages: generation of diversity (the construction of the initial agent society); primary selection (one or more selection mechanisms acting on agents that reduce initial diversity); basic mechanism of interaction (characteristic mechanism of the model through which model rules and interactions between agents); mechanism oriented to one or more strategies or one or more objectives; secondary selection (which focuses on the agent society resulting from the previous steps); final identification of the collective structure (stage where the properties and regimes resulting from the model dynamics are characterized).

Hamill and Gilbert (2016, pp. 237-238) present a basic framework for an agent-based model using Netlogo in a very didactical way. First, with a simple market model and, after that, introducing new features. It is a very useful tutorial to everyone who wants to make an agent-based model in macroeconomics. For these authors, what defines a good model is validation and that is not very simple to achieve in some cases: "(…) the importance of verification, that is, of ensuring that the program is doing what the modeler intended. That is clearly essential. But a bigger and more difficult question is: what makes a good model one? This assessment is called validation.” And they go on: “One approach is to measure the extent to which the observed macroeconomic data can be explained by micro level interactions. But that alone is not sufficient. To be good, an agent-based model must also make sense at the micro level, both in the characteristics of the agents and the manner in which they interact.”

Fagiolo and Roventini (2012, pp. 17-18) present the way a typical agent based model works based on the ten ingredients they described (see page 8). We will briefly describe the structure of the model. There is a population of agents (consumers, competing firms, households or traders, for example), that can be hierarchically organized and whose size even may change in time (population growth, for example). The evolution of this population and the whole system is observed in discrete time steps ($t = 1, 2, \ldots$), that can be minutes, hours, days, months, quarters or years. At each step t, every agent will be characterized by a number microeconomic variables $x_{i,t}$ (which may change) and by micro-economic parameters θ_i (fixed in time). And the economy itself, can have some macroeconomic (fixed) parameters Θ, for example interest rate level determined by the central bank, population growth assumptions or policy rules regarding migrations and borders openness degree. The models that will be presented in chapters 3, 4 and 5 have some of these macroeconomics fixed parameters.

Starting from initial conditions $x_{i,0}$ and micro and macro parameters define *ab inittio*, at each time step $t > 0$, the agents update their microeconomic variables according to the rules defined and this may happen in a random manner or can be triggered by the system. After the updating round involving the agents, their rules and their interactions, a new set of microeconomic variables is fed into the economy for the next step. The model can have, and normally have, stochastic component related with decision rules, interactions or expectations formation.

Marchi and Page (2014, pp. 7-8) describe the basics of the agent-based models in a similar manner. The agents are characterized by attributes that are the same used in game theory models such as beliefs, actions or payoffs, but can include the location of the agents too. Formally, we define set of attributes as:

$$\{X_{1,}X_2, \dots X_M\},$$

where each X_i is a countable set, i.e., $X_i = \{x_{i1}, x_{i2}, x_{i3}, \dots\}$.

To initiate the model, each agent attribute will have a value assigned. Attributes can change in time so, in formal terms, we introduce an index t to denote the state of an agent at time t. Considering a population of N agents, the state of an agent j at time t is given by:

$$a_j^t = (x_{j1}^t, x_{j2}^t, \dots, x_{jM}^t), \text{ with } x_{ji}^t \in X_i .$$

The configuration of the model, which refers to the collection of all the agent´s states, at time t is:

$$A_t = \{a_1^t, a_2^t, \dots, a_N^t\}(x_{j1}, x_{j2}, \dots, x_{jM}), \text{ with } x_{ji} \in X_i$$

Typically, according to these authors, an agent-base model can have between 2 and 10,000 agents depending on the objet modeled. Political models, for instance, can have few agents representing major parties but consumer models or migration models will have surely hundreds, thousands or even millions of agents. Researchers in agent-based modelling should bear in mind that simulations involving very large number of agents and periods can consume substantial computational time.

Gilbert (1999, pp. 9-24) presents some techniques to construct agents, learning processes, environments, memory, rules or to checking the code for error or to replicate experiments. Useful information for anyone interested in using these kind of tools.

Axelrod (2006) presents “a guide to newcomers to agent-based modeling in the social sciences” where he provides an extensive readings list with all the information need to concretize the task of using an agent-based model in social sciences. Axelrod (1997, pp. 206-221) provides some useful resources for agent-based modeling.

Delli Gatti, Desiderio, Gaffeo, Cirillo and Gallegati (2011, pp. 25-44) have a detailed description about "the making of a BAM [Bottom-up Adaptive Macroeconomics] model" which comprises the agent´s characteristics, the world in which they operate, their interaction and, of course, validation[19] and verification of the model. ODD protocol[20] is a common framework to describe agent-based models and can be used in every of its applications, independently of the area of research, because the processes have similar features. Normally, it includes an overview of the model (purpose, variables and schedule), some design concepts (about emergence properties, interaction or adaptation) and other details.

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[19] About empirical validation see also Delli Gatti, Desiderio, Gaffeo, Cirillo e Gallegati chapter IV (2011, pp.85-99)

[20] For a detailed description of the ODD protocol see Grimm, Berger, Bastiansen, Eliassen, Ginot, Giske, and Huth (2006) and Grimm, Berger, De Angelis, Polhill, Giske and Railsback (2010).

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3. INDIVIDUAL SATISFACTION AND ECONOMIC GROWTH IN AN AGENT-BASED ECONOMY[21][22]

Abstract

We combine macro and microeconomic perspectives in an agent-based endogenous growth model that uses individual satisfaction as a driver of human capital accumulation. The micro perspective is based on individual satisfaction: a utility function computed from the income variation in space (relative to others) and time. The macro perspective emerges from micro decisions that, at an aggregate level, determine an important social decision about the share of the working population engaged in producing ideas (i.e. skilled workers). Underlying our analysis is the Easterlin hypothesis (Easterlin 1974, 1995) which states that individuals care much more about their relative income than about increases in their own income, weakening the link between growth and income. Simulations show that growth and satisfaction levels are higher when relative and absolute incomes are equally weighted in satisfaction computation and are lower when satisfaction only depends on relative incomes.

Keywords: agent modeling, education, human capital, economic growth, individual satisfaction.

JEL codes: C63; E24; E70; O40

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3.1 Introduction

Agent-based modeling is a growing research area in economics (Kirman 2004; Tesfatsion 2006; Farmer and Foley 2009). Applications in macroeconomics, albeit increasing, are still relatively rare. There are, however, some examples, even hybrid approaches mixing traditional DGSE models with other non-standard characteristics (Lebaron and Tesfatsion 2008; De Grauwe 2010; Gati *et al* 2011). Financial crisis highlighted flaws in many models used in economic policy design, namely DGSE models. Homogeneity (representative agents) and rational expectations hypothesis are the two most criticized features. Agents are not, of course, homogeneous and economic reality is far more complex than this over simplistic assumption states. Rationality is also disputed by several neurological and psychological experiments (Kahneman 2003; Camerer, Loewenstein and Rabin 2011; Fehr and Rangel 2011).

In this paper, we use an agent-based model to assess the relation between individual satisfaction, economic growth and human capital accumulation. According to the so called Easterlin hypothesis (1995), individuals care much more about their income relative to others (relative in space) than about increases in income that go along with a general upward trend (relative in time). Layard, Mayraz and Nickell (2010), for example, find some empirical supporting evidence for the United States, Western Germany and other developed countries. This result have obvious implications in microeconomic decisions but can have also important macroeconomic consequences Some other researchers, however, do not share this extreme view, and the relative importance of absolute income for happiness is not at all settled in the literature (see Deaton (2008) for a discussion of the positive relationship between life satisfaction and national income).

This seemingly small distinction between two different kinds of happiness is of the utmost importance. It means that countries can experience income – or GDP – growth without corresponding increases in happiness levels. Blanchflower and Oswald (2004) documented it empirically for US and UK. Oswald (1997) reported happiness gains along with economic growth but “almost undetectable”. The same had concluded Frey and Stutzer (2000). Kahneman *et al* (2005) also raise doubts about the income-happiness link.

On the contrary, Stevenson and Wolfers (2008) find no threshold beyond which wealthier countries would experience no further increases in satisfaction. Frey and Stutzer (2002) argue that happiness has quite relevant implications for both theory and economic policy. This because it is a variable that does not correspond to the concept of utility used in many economic models and can have consequences in different aspects of the economic activity, namely in economic growth. Di Tella, MacCulloch and Oswald (2003) presented the "macroeconomics of happiness". Luttmer (2004) concludes that relative incomes are important for happiness. In a panel data analysis, it confirms the idea that "lagging behind the Joneses" is relevant for individual satisfaction levels. Carbonell (2005) find that the income of the reference group is almost as important as the own income for individual happiness.

We use a model derived from Jones (2005) with an economy with skilled and unskilled workers in an overlapping generation [23]environment to assess this Easterlin hypothesis and its impact on economic growth and human capital accumulation. Several endogenous growth models emphasize the role of "ideas" in economic growth. In this model, ideas are produced by a fraction of the working population - the skilled workers - and are used by the rest of the workers - the unskilled workers- to produce final goods. An agent decision to study is taken following a socially conditioned economic reasoning based on his or her individual satisfaction perspectives. Each agent decision will be based on the satisfaction level of his neigbor which, in turn, depends largely on the relative position of the agent´s income in space (compared to others) and in time (variations of his own income).

This satisfaction-based education decision and its impact on economic growth is evaluated for different scenarios, based precisely on different weights given to individual relative income in space and time. Scenarios are tested against each other in terms of long term growth and satisfaction. Our results indicate that when personal wellbeing - i.e. satisfaction - depends exclusively on interpersonal comparisons – satisfaction becoming a kind of rival good - the economy grows less and, at the local level, there is almost no clustering between skilled and unskilled workers.

[23] It is an extension of Araújo and St. Aubyn (2008) and Martins, Araújo, Santos and St. Aubyn (2009) models using individual satisfaction as the key variable. In what follows, equations (1) to (9) are very similar or equal to the corresponding ones in those models.

3.2 The model

3.2.1 Population

Our economy has N agents: $N/2$ junior and $N/2$ senior. Each agent lives for two periods. Population size does not change and generations overlap. A young agent can be either a student or an unskilled worker. Thus, population has always four groups: young students; junior unskilled workers; senior unskilled workers (those that did not study in the previous period) and skilled workers.

3.2.2 Space and decision to educate

There is a neighbor effect in the education decision. We can assume that, *ceteris paribus*, a children's education attainment depends positively on the average human capital stock in his or her neighborhood. In our model, the decision is based on the observed relative satisfaction in skilled and unskilled workers. More precisely, is based on the number of satisfied skilled workers and satisfied unskilled workers on his neighbourhood defined on a ring with a neighbourhood size 2g. In formal terms, agents decide to study if:

$$nS_t^s > nS_t^u \quad (1)$$

where nS_t^s and nS_t^u are respectively, the number of satisfied skilled workers and satisfied unskilled workers in the neighbourhood (i.e. agents with positive satisfaction levels).

3.2.3 Production

Production is computed from the stock of ideas and from the unskilled labor supply. Unskilled workers provide regular work while skilled workers produce ideas. Production is defined as:

$$Y_t = A_t U_t + \varepsilon_t \quad (2)$$

where Y_t is production in period t , A_t the stock of ideas in period t , U_t the number of unskilled workers in period t and ε_t a productivity shock (with uniform distribution between -0.5 and 0.5) in period t. The evolution of the stock of ideas is given by:

$$\Delta A_t = A_{t-1}\delta S_t + \gamma D_t \quad (3)$$

where S_t represents skilled labor, δ is a parameter related with marginal productivity of skilled labor, D_t is a measure of distance between skilled workers and γ a parameter of the strength of the team effect. This means that production of ideas is higher when skilled workers are closer to each other and in the presence of a higher team effect. D_t is defined as:

$$D_t = \frac{1}{S_t}\sum_{i,j=1}^{S}\frac{1}{|i-j|} \quad (4)$$

with $i \neq j$ and being thus smaller when skilled workers are located far from each other and larger in the opposite situation. i and j are the positions of agents i and j in the ring.

3.2.4 Income distribution and wages

Production in each period is divided between skilled and unskilled workers. In mathematical terms:

$$Y_t = Y_t^U + Y_t^S \quad (5)$$

where Y_t^U denotes the total income of unskilled workers and Y_t^S the total income of skilled workers.

The income distribution – the social contract in this society – specifies that skilled workers receive the share related to the production of ideas and unskilled workers receive what would have been produced if ideas remained constant. Thus, the unskilled workers income is computed considering the previous period stock of ideas and all the additional income due to new ideas belongs to skilled workers. Productivity shocks are shared, in equal parts, by skilled and unskilled workers. The total income for unskilled (Y_t^U }) and skilled workers (Y_t^S) is given by:

$$Y_t^U = A_{t-1}U_t + \frac{\varepsilon_t}{2} \quad (6)$$

$$Y_t^S = (A_t - A_{t-1})U_t + \frac{\varepsilon_t}{2} \quad (7)$$

Wages per worker are determined dividing the total income by the total number of skilled (L_t) and unskilled workers (U_t):

$$w_t^U = \frac{A_{t-1}U_t}{U_t} + \frac{\varepsilon_t}{2U_t} = A_{t-1} + \frac{\varepsilon_t}{2U_t} \quad (8)$$

$$w_t^S = \frac{Y_t^S}{L_t} + \frac{\varepsilon_t}{2L_t} = (A_t - A_{t-1})\frac{U_t}{L_t} + \frac{\varepsilon_t}{2L_t} \quad (9)$$

3.2.5 Satisfaction

Satisfaction is a measure of individual well-being that comprises relative position of the agent's income in space and in time but also takes into account the initial expectations of the agents when they decide about education. Initial expectations are randomly generated and they can be interpreted as a sort of fixed cost. The higher the expectation level of the agent, the harder the possibility of becoming satisfied with the options he made. Educated workers are provided with some amount of satisfaction (α)

for the simple fact of being educated. Therefore, and respectively for skilled and unskilled workers, individual satisfaction is computed as:

$$F_{i,t}^{S} = \alpha - c_i + (1-\omega)\left(w_t^S - \frac{w_t^U}{\beta(\rho)}\right) + \omega(w_t^S - w_{t-1}^S) \tag{10}$$

$$F_{i,t}^{U} = -c_i + (1-\omega)\left(w_t^S - \frac{w_t^U}{\beta(\rho)}\right) + \omega(w_t^U - w_{t-1}^U) \tag{11}$$

where c_i is the initial expectation of agent i, taking values in the interval [-0.5,0.5][24] . w_t^S and w_t^U represents, respectively, skilled and unskilled wages. α is an exogenous parameter, ρ is a discount rate and $\beta(\rho)$ a monotonic function with $\frac{\delta\beta}{\delta\rho} > 0$ that is used to compare present values of skilled and unskilled workers future incomes[25]. ω is a parameter taking values between 0 and 1 that represents the weighted relative income (in space) and income growth (in time). When $= 0.5$, both relative incomes have equal weight. Setting ω with different values allows for balancing the range of influences that determines individual satisfaction. This parameter is used to configure the three scenarios presented in the following section.

The Easterlin idea is modelled as the relative weight between cross section and time series income comparisons, i.e., ω. The Easterlin paradox would hold when ω becomes close to 0. In that case, income growth would not provide any satisfaction. Parameter c_i only purpose is to introduce some individual heterogeneity.

[24] Biondo, Pluchino and Rapisarda (2012) presents a ABM model using an expectation component used for agent emigration decision that depends that tends to disappear over time after the migration

[25] $\beta(\rho)$ is given by $\beta = \frac{(1-\rho)^9-(1+\rho)^{8-\tau}}{1-(1+\rho)^{8-\tau}}$. τ represents the number of years to the end of active life and was set to 48. We considered that skilled agents spend nine more years at school than unskilled agents. When the discount rate is higher, the future is less valued and therefore the skilled labour wage has to be higher for agents to become indifferent between acquiring skills through education and to remain unskilled. $\beta(\rho)$ function is derived in the Appendix.

3.3 Results

When the economy starts, there are 50 unskilled agents, junior or senior. The other 50 individuals are either students, if they are junior, or skilled employees, if they are senior. In a typical, average, baseline simulation, the number of employees equals 75, 25 being skilled. The location (on the ring) of each agent, junior or senior and skilled or unskilled, is randomly determined. The neighbourhood range was set to 3 ($g = 3$), meaning that when an agent decides whether to pursue his or her studies by considering the satisfaction of his or her six closest senior neighbors (three to the left and three to the right). According to equation (1), the agent will chose to become educated if the number of satisfied skilled workers exceeds the number of satisfied unskilled workers. The discount rate equals five percent ($\beta = 0.05$), a value not very different from empirically observed real interest rates.δ

Simulations were performed for three scenarios of parameter ω: 0, 1 and 0.5. All scenarios start with 100 agents (50 skilled and 50 unskilled workers; 50 junior and 50 senior), for nine generations ($R = 9$), with no team effects ($\gamma = 0$). Satisfaction in the first period ($t = 1$) is randomly set for all agents with values between -0.5 and 0.5. As they did not have wages in the previous period, junior students 'satisfaction is derived from senior workers satisfaction - it is randomly generated with values ranging between the maximum and the minimum values among senior workers. Stock of ideas $A(1)$ is initialized with a value above 0 determined by the average distance between educated workers in the initial spatial distribution. The skilled labour productivity parameter $\delta$was set to 0.025 and the education satisfaction parameter α was set to 1.

Table 3.1 shows the model parameters values in each scenario:

	N	R	δ	α	γ	ρ	g	$U_{i,t=0}$	ω
Baseline									**0.5**
1	**100**	**9**	**0.025**	**1**	**0**	**0.05**	**3**	**50**	**0**
2									**1**

Table 3.1: Simulation scenarios

Reported results are the average results after 1000 simulations. The baseline scenario is the best one in terms of economic growth. But, on the contrary, educated workers satisfaction is slightly lower than in Scenario 3. Satisfaction and growth have the worst performances in Scenario 2, the scenario where only relative income (in space) matters. This is also the least clustered society, ending up with (on average) 33 partitions.

It is possible to evaluate the degree of clustering (skilled/non-skilled) in this economy by counting the number of observed partitions. A small number of partitions corresponds to high local clustering. The final number of partitions for each scenario is presented in Table 3.2. The initial average number of partitions is 50. Final U and Final S indicate the final number of unskilled and skilled workers. $F^U$and F^S represent, respectively, the percentage of satisfied non-educated and the percentage of satisfied educated agents.

	Y growth	*Final U*	*Final S*	F^U *(%)*	F^S *(%)*	*Final partitions*
Baseline	**0.65**	**50**	**25**	**100**	**98.7**	**6.6**
1	**0.61**	**46**	**23**	**78.7**	**21.3**	**33**
2	**0.64**	**43**	**29**	**100**	**100**	**6.3**

Table 3.2: The outcomes of the three scenarios after 1000 simulations

Figure 3.1 depicts initial distribution of skilled and unskilled workers and also final partitions in each of the scenarios.

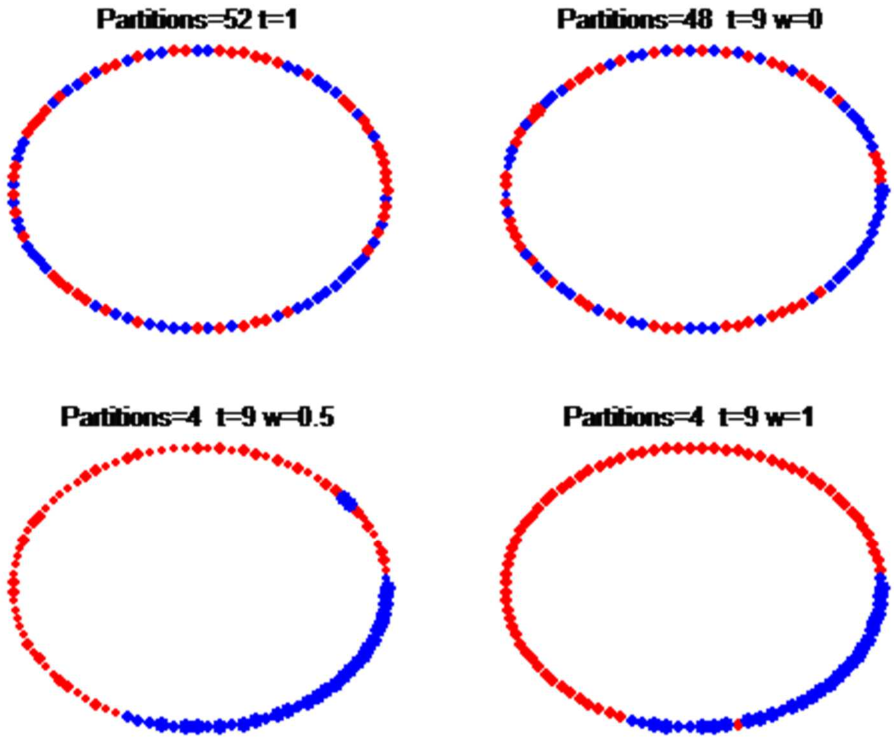

Figure 3.1: The first plot shows the initial distribution of skilled and unskilled agents on a ring and the corresponding number of partitions of a typical run in any of the three scenarios. The second, third and fourth plots show the final distributions and the number of partitions in each different scenario. The size of the nodes is proportional to its satisfaction and the color identifies skilled (blue) and unskilled workers (red).

It shall be noticed that in any of the three scenarios and at each time step (each generation), there is no direct interaction between the agents which, instead, react to collective variables (local and global), that they themselves create with their individual decisions. The dynamics of the model has two main mechanisms: a local mechanism that operates when the agents decide about education - where the collective variables are the number of satisfied-skilled and satisfied-unskilled workers in the neighborhood; and a global mechanism – operating when accounting for individual satisfaction - where the collective variables are the wages of skilled and unskilled workers. In this context, the interaction through global variables (wages) operates faster in the overall dynamics of the model. Simultaneously, the interaction through local variables (individual satisfaction) gives rise to a slower contribution, i.e., a contribution whose consequences are not immediately incorporated, since changes in individual satisfaction will

affect the decisions of the next generation[footnote: The field of dynamical systems or, more precisely, its contributions to the understanding of the interplay of local and global variables (see for, instance, Vilela Mendes (2001)) informs that in some systems, the essential mechanism driving the overall dynamics of the system is the slow dynamics, whereas the fast dynamics operates only as a background which is selected by the slow evolution. Our results are in line with the consequences of the above described interplay between local and global interactions. When personal wellbeing depends exclusively on the influence of interpersonal comparisons, there is almost no clustering as the way the agents organize themselves in space (number of partitions) approaches the random (initial) situation.
This is due to the fact that when the influence of interpersonal comparisons dominates, the slow dynamics depending on a rival good drives the set of agents to an unstable situation in what concerns their satisfaction-based education decision. In this setting, neither local clustering nor any structural organization happens to take place.].

3.4 Conclusions

Agent-based modeling and endogenous growth are combined in a model that uses individual satisfaction as a driver of human capital accumulation. It is a macro model with micro foundations in an overlapping generation environment, where agents decide to study based on individual satisfaction of their peers (neighbors). Satisfaction is a kind of a utility function with two main pillars - relative income (skilled versus unskilled workers) and the evolution of income in time - and also the initial expectations of agents. We simulate three scenarios weighting differently the two main pillars of individual satisfaction in order to assess the Easterlin paradox. The baseline scenario, where both measures of income have equal weights, displays the best performance in the long run growth. Moreover, this scenario is characterized by a high level of local clustering. When only relative income matters for satisfaction, growth and local clustering are lower - the idea behind Easterlin hypothesis. In the other extreme scenario, when only income growth matters, local clustering and growth are similar to the baseline outcomes.

Future work will deal with several improvements of the model specification in terms of extension (more than one economy and population dynamics), agent´s decisions (return to school after some years in labor market or having different education degrees) and heterogeneity (different individual parameters α and ω or labor specialization).

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3.6 ODD protocol

3.6.1 Overview

Purpose: Our model, derived from Jones (2005), represents an economy with skilled and unskilled workers in an overlapping generation environment and aims to show how individual satisfaction can be a driver of long term economic growth. More concretely, this means to observe the economic consequences of individual decision about studying based on satisfaction level of his neighbors which, in turn, depends largely on the relative position of the agent´s income in space (compared to others) and in time (variations of his own income).

State variables and scales: Agents are defined by two different characteristics – age (junior or senior) and education (skilled and unskilled)

– and with a heterogeneous initial expectation parameter c_i related with their satisfaction.

The state variables are:

- $\boldsymbol{\omega}$ (relative weight given to absolute income in satisfaction)
- $\boldsymbol{\delta}$ (related with marginal productivity of skilled labor)
- $\boldsymbol{\gamma}$ (strength of the team effect)
- $\boldsymbol{g}$ (relevant neighbourhood range)
- $\boldsymbol{\rho}$ (intertemporal discount function parameter)
- α (education satisfaction parameter)

Process overview and scheduling: In each period, the junior agents decide about studying based their neighbors´ satisfaction which, in turn, depends on relative income in space and time. The agents that decided not to study will remain unskilled in the older period of their life.

3.6.2 Design concepts

Emergence: Individual and local decisions about education have global consequences in terms of economic growth and clustering, depending on the assumptions considered in each scenario.

Adaptation: Individual agent characteristics don´t change and parameters are, therefore, constant.

Fitness: Economic performance is evaluated based on economic growth and clustering.

Prediction: Agents face no uncertainty in their decisions.

Interaction: Neighbourhood effects are crucial for the dynamics of the model. Each individual decision is based on the peer influence.

Stochasticity: The model has an individual stochastic characteristic related with the expectations of each agent as a component of his satisfaction. Initial satisfaction is randomly generated. Production function has also a random error term.

Collectives: Skilled versus non-skilled is most relevant collective analysis.

Observation: Economic performance (numerically and graphically) and clustering (depicted in a ring).

3.6.3 Details

Initialization: The initial characteristics of the agents are randomly generated: junior and senior, skilled and unskilled, and individual expectation parameter. Satisfaction in the first period is randomly set for all agents with values ranging between -0.5 and 0.5. Stock of ideas $A(1)$ is initialized with a value above 0 determined by the average distance between educated workers in the initial spatial distribution.

Input: Simulations for three scenarios of the parameter ω: 0, 1 and 0.5. Starting with 100 agents (50 skilled and 50 unskilled workers; 50 junior and 50 senior), for nine generations ($R = 9$), with no team effects ($\gamma = 0$). Skilled labor productivity parameter δ was set to 0.025 and the education satisfaction parameter α was set to 1.

3.7 Appendix 1: $\boldsymbol{\beta}(\boldsymbol{\rho})$

At the beginning of period t , an agent has perfect knowledge of period $t-1$ wages, namely w_{t-1}^{S} and w_{t-1}^{U}, the skilled and unskilled labour wage, respectively. Assume that agents take these values as the ones that will prevail in the future, and, for the sake of simplicity, denote them by w^{S}} and w^{U}.

Suppose skilled agents spend nine more years at school than unskilled agents. For example, one can think they spend two more years at secondary school, four additional years to take a first degree, and finally three more years in some form of post-graduate studies. PVE, the present value of future wages for an agent that is starting his or her education to become skilled is then:

$$PVE = w^{S}[1 + (1+\rho)^{-9} + (1+\rho)^{-10} + \cdots + (1+\rho)^{\tau}] \qquad (12)$$

where ρ is a time preference rate or a discount rate, and τ is the number of years to the end of active life, likely to be comprised between 45 and 50.

At the same time a future skilled agent starts his or her education, unskilled agents start working. With the hypothesis above, this means they work nine more years when compared to skilled workers. Let PVU be the present value of all wages earned by unskilled workers:

$$PVU = w^U[1 + (1+\rho)^{-1} + (1+\rho)^{-10} + \cdots + (1+\rho)^{\tau}] \qquad (13)$$

Comparing equations (12) and (13), it is apparent that w^S must be greater than w^U for there to be any chance of PVE being greater than PVU. In this case, and from a pure income perspective, i.e., taking aside any subjective preference for education, the agent would chose to proceed into further education and not to remain unskilled. Let β be the ratio between w^S and w^U that makes the present value of skilled labour wages equal to the present value of unskilled labour wages:

$$\frac{w^S}{w^U} = \beta = PVE = PVU \qquad (14)$$

From equations (12), (13) and (14), it gives:

$$\beta = \frac{1+(1+\rho)^{-1}+(1+\rho)^{-10}+\cdots+(1+\rho)^{\tau}}{1+(1+\rho)^{-9}+(1+\rho)^{-10}+\cdots+(1+\rho)^{\tau}} = \frac{A}{B} \qquad (15)$$

with $A = 1 + (1+\rho)^{-1} + (1+\rho)^{-10} + \cdots + (1+\rho)^{\tau}$ and $B = 1 + (1+\rho)^{-9} + (1+\rho)^{-10} + \cdots + (1+\rho)^{\tau}$. It is easy to show that:

$$A = \frac{1+\rho-(1+\rho)^{-\tau}}{\rho} \qquad (16)$$

$$B = \frac{(1+\rho)^{9}-(1+\rho)^{8-\tau}}{1-(1+\rho)^{8-\tau}} \qquad (17)$$

Replacing A and B in expression (15) and simplifying, it gives:

$$\beta = \frac{(1+\rho)^9 - (1+\rho)^{8-\tau}}{1-(1+\rho)^{8-\tau}} \tag{18}$$

Note that β approaches $(1+\rho)^9$ when τ tends to infinity, and that β is an increasing function of ρ.

When the discount rate is higher, the future is less valued, and therefore the skilled labour wage has to be higher for agents to become indifferent between acquiring skills through education and to remain unskilled. In our simulations, we set τ =48.

3.8 Appendix 2: Matlab

3.8.1 The main model

```
function [resp] = brain
(N,V0,E,T,Lo,R,delta,disc,factor,team,viz,fig,banco,omega);

% number of lines to the plot
% N =number of agents, always pairs (100)
% V0 = number of older agents(50)
% E = number of skilled agents   (50)
% T = number of agents type 1   (50)
% Lo = number of local agents   (50)
% R = number of periods      (9)
% delta = marginal productivity of skilled
% factor = wage relevance in education decision (1)
% disc = discount rate (0.05)
% banco= monetary policy parameter (0.5)
% team = team effect (0)
% viz = neighbourhood range (3)
% Y keeps production in each period

% resp=BrainMar2016(100,50,50,50,50,9,0.09,0.05,1,0,3,1,0,0.5);
% chamada com a baseline

% Initiation I

A1=zeros(1,R);   A2=zeros(1,R);   WE1=zeros(1,R);   WE=WE1;   WE2=WE1;
WnE=zeros(1,R); L=zeros(1,R); LI1=L; LI2=L;
Y=zeros(1,R);  Y1=zeros(1,R);  Y2=zeros(1,R);  M=ones(N,4);   M(:,2)=-1;
M(:,3)=-1; i=zeros(1,R); CH=i;
```

```
numerador=(1+disc)^(9)-(1+disc)^(-40);
denominador=1-(1+disc)^(-40);

compare = numerador/denominador; % computed from discount rate

Mrand=randperm(N);
M(Mrand(1:V0),1)=zeros(V0,1);   % age
Mrand=randperm(N);
M(Mrand(1:E),2)=ones(E,1);      % Education
Mrand=randperm(N);
M(Mrand(1:T),3)=ones(T,1);      % Type
Mrand=randperm(N);
M(Mrand(1:Lo),4)=ones(Lo,1);    % Localization

S=zeros(N,R)+1;

% Column 1 M - senior (=0) junior (=1)
% Column 2 M - skilled (=1) unskilled (=-1)
% L  = number of unskilled
% LI = number of skilled senior (producing ideas)
% VF = average distance between skilled senior

% Initiation II

   M0=M;
   RwE=1; % relation between wages weighted by factor
   Segrega=SegBrain(M,N,fig);
   F1=find(M(:,2)==1 & M(:,1)==1 & M(:,3)==-1);
   F2=find(M(:,2)==1 & M(:,1)==1 & M(:,3)==1);
   VF1(1)=CalcDistSeniors(M,N,-1); % run program and compute distances
   VF2(1)=CalcDistSeniors(M,N,1);

A1(1)=delta*length(F1)+team*VF1(1);A2(1)=delta*length(F2)+team*VF2(1);
A(1)=A1(1)+A2(1); % Ideas initiated with distance seniors
WnE(1)=0;    L(1)=sum(M(:,2)==-1);    %    unskilled    have    wage=0
WE1(1)=A1(1);WE2(1)=A2(1);WE(1)=WE1(1)+WE2(1);
YE1(1)=A1(1)*L(1); YE2(1)=A2(1)*L(1);YE(1)=YE1(1)+YE2(1);

% Cicle

for z=2:R

    M(:,1)=mod(M(:,1)+1,2);% júnior became sénior and newborn junior
replace sénior
    Rel(z)=RwE*factor/compare;

    L(z)=sum(M(:,2)==-1);               % number of unskilled
    LI(z)=sum(M(:,2)==1 & M(:,1)==1);   % number of senior skilled

    M=DecEduca(M,N,L,LI,Rel(z),viz);    % run program education
    L(z)=sum(M(:,2)==-1);

     LI(z)=sum(M(:,2)==1 & M(:,1)==1);

     LI1(z)=sum(M(:,2)==1 & M(:,1)==1 & M(:,3)==-1);   % number senior
skilled type 1

     LI2(z)=sum(M(:,2)==1  & M(:,1)==1& M(:,3)==1);       % number of
skilled senior type 2
     VF1(z)=CalcDistSeniors(M,N,-1);                  % run program to
compute distances
    VF2(z)=CalcDistSeniors(M,N,1);VF(z)=VF1(z)+VF2(z);
    % Find skilled, compute ideas
    F1=find(M(:,2)==1 & M(:,1)==1 & M(:,3)==-1);
```

```
    F2=find(M(:,2)==1 & M(:,1)==1 & M(:,3)==1);
    CH(z)=2*rand-1;% random shock
    i(z)=banco*(CH(z-1)+i(z-1)); % interest rate corresponding to ½
output gap in previous period          A1(z)=A1(z-1)+A1(z-
1)*(delta*length(F1)+team*VF1(z));
    A2(z)=A2(z-1)+A2(z-1)*(delta*length(F2)+team*VF2(z));
    A(z)=A1(z)+A2(z);
    % Calcula Produto
    YnE(z)=A(z-1)*L(z)+(CH(z)+i(z))/2;  % total wages unskilled
YE1(z)=(A1(z)-A1(z-1))*L(z)+(CH(z)+i(z))/2;   % total wage skilled 1
YE2(z)=(A2(z)-A2(z-1))*L(z)+(CH(z)+i(z))/2;   % total wages skilled 2
YE(z)=YE1(z)+YE2(z);
    Y(z)=YnE(z)+YE(z);
    % compute wages
    WnE(z)=YnE(z)/L(z);     %     total     wages     unskilled
WE1(z)=YE1(z)/LI1(z);  % total wage skilled 1
    WE2(z)=YE2(z)/LI2(z);  % total wages skilled 2
    WE(z)=WE1(z)+WE2(z);
    if (L(z-1)>0 & LI(z-1)>0 & (N-L(z-1))>0) RwE=(WE(z-1)/(LI(z-
1))/(WnE(z-1)/L(z-1))); else RwE=1; end
    NF=find(M(:,2)==-1); F=find(M(:,2)==1);        % find unskilled
and senior skilled
    % Compute satisfaction
    c=rand(size(S(F,z),1),1);
    s1=sign(WE(z)/LI(z)-(WE(z)+WnE(z))/(L(z)+LI(z)));
    s2=sign(WE(z)/LI(z)-(WE(z-1)/LI(z-1)));
    S(F,z) =-c+S(F,z-1)+2*(1-omega)*s1+2*omega*s2;     % update senior
skilled satisfaction
    c=rand(size(S(NF,z),1),1);
    s3=sign(WnE(z)/L(z)-(WE(z)+WnE(z))/(L(z)+LI(z)));
    s4=sign(WnE(z)/L(z)-(WnE(z-1)/L(z-1)));
    S(NF,z)=-c+S(NF,z-1)+2*(1-omega)*s3+2*omega*s4;        % update
unskilled satisfaction
end

Segrega=SegBrain2(M,S(:,z),2*fig,R); % run program segregation

MS=mean(S);

% output matriz "resp", each run or period per line

resp(:,1)=(A/A(1))';resp(:,2)=Y'/N;resp(:,3)=WnE';resp(:,4)=WE';resp(:
,5)=(Y/Y(1))';resp(:,6)=L';
resp(:,7)=LI';resp(:,8)=Segrega';resp(:,9)=VF';resp(:,10)=sum(S(NF,:)>
0);resp(:,11)=WE1';resp(:,12)=WE2';
resp(:,13)=LI1';resp(:,14)=LI2';resp(:,15)=VF1';resp(:,16)=VF2';resp(:
,17)=Rel';resp(:,18)=sum(S(F,:)>0);

if fig>0
   figure;
         subplot(3,1,1);plot((S(F,1:end)'));title(['Evolution      of
Satisfaction    Educated    -    \omega='    num2str(omega)]);
ylabel('S_i');xlabel('t');grid
         subplot(3,1,2);plot((S(NF,1:end)'));title('Evolution      of
Satisfaction non-educated'); ylabel('S_i');xlabel('t');grid
         subplot(3,1,3);plot(MS(1:end));title('Evolution  of  <S>');
ylabel('<S>');xlabel('t');grid
   figure;

plot((1:z),resp(:,2),'b',(1:z),resp(:,4),'g:',(1:z),resp(:,3),'r:',(1:
z),resp(:,11),'c',(1:z),resp(:,12),'m:', ...

(1:z),resp(:,2),'b*',(1:z),resp(:,4),'g*',(1:z),resp(:,3),'r*',(1:z),r
esp(:,11),'c*',(1:z),resp(:,12),'m*');
```

```
        title(['N='  num2str(N)  '    E_i=  '  num2str(E)  '    E_f=  '
num2str(sum(M(:,2)==1))  '   SE=  ' num2str(sum(M(:,1)==1))  ' \delta='
num2str(delta) ' - \omega=' num2str(omega)]);
        legend('Y_{t}/N', 'WE_{t}', 'WnE_{t}','WE1_{t}', 'WE2_{t}',0);
        xlabel(['t '  ]);
end
```

3.8.2 Segregation

```
function [sas] = Segregation (M,S,fig,R);

% vector M with N linhas e 1 column
% value indicates the element type % questao)
% this program depicts graphically a ring with both types of elements:
full and empty
N=size(M,1);
FT=(1:N);
Fe1=find(M(:,2)==1 & M(:,3)==-1);
Fe2=find(M(:,2)==1 & M(:,3)==1);
Fne=find(M(:,2)==-1);
for        roda=1:N-1        segrega(roda)=M(roda,2)+M(roda+1,2);end;
segrega(N)=M(N,2)+M(1,2);

tet=linspace(0,2*pi,N);
ss=sin(tet);a1=0;a2=a1;a3=a1;
cc=cos(tet);
if fig>0
    figure(fig);hold on;
    for x=1:length(Fne) if S(Fne(x))>0 tam=8*S(Fne(x))+15; a1=a1+1; else
tam=15; end; scatter(cc(Fne(x)),ss(Fne(x)),tam,'ro','filled'); end;
    for x=1:length(Fe1) if S(Fe1(x))>0 tam=8*S(Fe1(x))+15; a2=a2+1; else
tam=15; end; scatter(cc(Fe1(x)),ss(Fe1(x)),tam,'ko','filled'); end;
    for x=1:length(Fe2) if S(Fe2(x))>0 tam=8*S(Fe2(x))+15; a3=a3+1; else
tam=15; end; scatter(cc(Fe2(x)),ss(Fe2(x)),tam,'bo','filled'); end;
    hold off; axis off
    sas=N-(sum(abs(segrega(:))))/2;
    title(['Partitions : ' num2str(sas) '  t=' num2str(R)])
end
     sas=N-(sum(abs(segrega(:))))/2;
% sas returns the number of partitions in the ring
```

3.8.3 Education decision

```
function [M]=DecideEduca(M,N,L,LI,Rel,viz);

        kmax=0;
        ME=find(M(:,1)==0); % find newborn with type determined   by
rWE/WnE
        LME=size(ME,1);     % number of newborn
for x=1:LME            % each newborn computes the main type in his
neighbourhood
             k=viz;          % viz is na exogenous parameter (agent´s
"myopia")
```

```
            M(ME(x),2)=0;      %  agent  himself  is  not  considered
vizinhos=0;
            Aux1=mod(ME(x)-k,N);  if  Aux1==0  Aux1=1;end;  %  solve
frontier conditions
            Aux2=mod(ME(x)+k,N); if Aux2==0 Aux2=1;end;
            if Aux1<Aux2 vizinhos=M(Aux1:Aux2,2).*M(Aux1:Aux2,1);
               else              vizinhos=[M(Aux1:N,2)'.*M(Aux1:N,1)'
M(1:Aux2,2)'.*M(1:Aux2,1)'];end; % avoids neighbourhood beyond 100

while sign( Rel*sum(vizinhos==1)-sum(vizinhos==-1) )==0 & k<N/2       %
alarga enquanto empatado
                   k=k+1;
                   Aux1=mod(ME(x)-k,N); if Aux1==0 Aux1=1;end;  % solve
frontier conditions
                   Aux2=mod(ME(x)+k,N); if Aux2==0 Aux2=1;end;
                   if                                          Aux1<Aux2
vizinhos=M(Aux1:Aux2,2).*M(Aux1:Aux2,1); % new neighbors do not count
because their product is zero
                   else          vizinhos=[M(Aux1:N,2)'.*M(Aux1:N,1)'
M(1:Aux2,2)'.*M(1:Aux2,1)'];end;  %  avoids  neighbourhood  beyond  100
end
               if k>kmax kmax=k;end; Gk=kmax;
           M(ME(x),2)=sign(Rel*sum(vizinhos==1)-sum(vizinhos==-1)   );
% determine main type of the agent
        end
```

3.9 Discussion

This model is an extension of the agent-based model presented by Araújo and St. Aubyn (2008) which, in turn, adapts Jones (2005) endogenous growth model. In the original model, Araújo and St. Aubyn, which are also co-authors of this paper and this thesis supervisors, used the model to test how different education endowments can produce different economic outcomes and how social interactions are vital to human capital accumulation and economic growth.

In this model, the most important innovation is the introduction of the individual satisfaction. Agent´s decision about studying is based on his peer satisfaction that is based on two different components: an absolute income (and its evolution in time) and a relative income (compare with other agents). Our aim was to test the Easterlin hypothesis, i.e., the relation between income and happiness is non-linear. Agent´s might prefer relative income instead of absolute income and, given different assumptions about these parameters, long-term economic growth can be affected.

We simulate three scenarios weighting differently relative and absolute income. The baseline scenario, where both have equal weights, displays the best performance in the long run growth and we find a high level of local clustering. When only relative income matters, growth and local clustering are lower. In the other extreme scenario, when only income growth matters, local clustering and growth are similar to the baseline.

The paper was published in Computational Economics (2018) after some revisions and improvements, namely more details about the functioning of the model itself and how the Easterlin paradox was tested. It is a contribution to agent-based models application in macroeconomics, in particular, in endogenous growth models with heterogeneity and stochastic features.

4. SOVEREIGN DEFAULT CONTAGION AND MONETARY POLICY IN AN AGENT-BASED MODEL[26]

Abstract

Sovereign default contagion was one of the most debated topics during the Eurozone sovereign debt crisis. Despite all the improvements in the financial situation since 2010, in particular after European Central Bank non-conventional monetary policy instruments, the roots of the problem and policy prescriptions to deal with it are still under dispute today. Using an agent-based model, we simulate sovereign default contagion for different monetary policy stances in a world where countries have random incomes, heterogeneous borrowing behaviors and risk aversion levels and where governments can enter in *ex-ante* agreements to protect against default. We conclude that default contagion can be a very fast and 'destructive' process, higher risk aversion levels are associated with lower default rates and monetary policy can have a very important role preventing sovereign defaults through zero interest-rate and quantitative easing policies.

JEL: C63 E62 G01

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4.1 Introduction

During the first ten years of European Monetary Union (EMU), government bond yields in Eurozone countries were almost the same. This was a surprising and a worrying behavior of the investors: every country with a German sovereign risk premium level, independently of its specific economic and fiscal fundamentals. The financial crisis triggered a review of these risk assessment procedures, in particular after the first Greek problems in late 2009.

Hesitations, sloppy political reactions and very high debt-GDP ratios were the perfect ingredients to the financial turmoil that followed: four bailed-out countries (Portugal included), several new instruments created to strengthen EMU firepower against financial turbulence and a ‘real’ economy trying to recover several years after 2009´s Great Recession. In some countries, like Portugal, real GDP in 2017 it is still below its 2008 level.

On the stage of the Eurozone sovereign debt crisis debate are two opposing political views about the nature of the problem and its solutions: is it simply a liquidity crisis that should be solved by liquidity providing mechanism such as the ECB´s long-term refinancing operations or the more recent asset purchase program?; or, on the contrary, is it a solvency problem and countries should embark in fiscal tightening policies to assure the investors that debt will be fully payed?

The truth is probably somewhere in the middle. These kind of sudden stop phenomena in debt markets have deep roots on the short-term fluctuations of the market sentiment but have also less immediate causes related to the fundamentals of the economies. Investors awakened, in the blink of an eye, from a dream where all the countries were equal to find that the differences exist and they are huge. As a consequence, they were frightened and run, as fast as they can, to safe havens inside and outside Eurozone, like the German Bunds or Swiss assets.

Part of the blame should be given to the EMU design itself. Namely, the inexistence of mechanisms to deal with this sort of turbulence and to cope with asymmetric shocks in general.

The technicalities about the intervention mechanisms are, probably, the easiest part to deal with. The hardest part was always to have enough agreement between the governments. Because the diagnosis was often blurred by simplistic formulations such as creditors *versus* debtors; hard working countries *versus* lazy countries or spenders *versus* savers. Different concepts of guilt also played an important role in the discussion. Some countries believe that the 'crisis countries' were responsible for their own faith and should pay to avoid moral hazard. Other, instead, think that this is a systemic problem that must be addressed in a systemic way and not by simply putting all the effort in some countries.

In fact, beyond further considerations about solidarity or Europeanism, there are good reasons for a country to share the default risk of its neighbors. First, because it is probably affected by the default of its neighbor. This can happen directly - if the country (its companies, households or even public entities) is a net creditor of its neighbor debts – or indirectly by traditional economic and financial linkages.

Even for very large and resilient economies, it is not very likely that a 'bad neighbor' has no consequences at all. Financial and economic linkages, such as trade or investment (direct investment and portfolio investment), tend to be stronger for geographically closer countries.

At the same time, regions are frequently considered – in the perspective of large global investors, for instance –almost homogeneous blocks. The Asian crisis in late 90´s is a very good example of how a financial turmoil can spread rapidly between neighbor countries, due precisely to the fact that large institutional investors – mutual funds, in particular – withdraw their investments in block from that region.

The second good reason for sharing the default risk is the possibility of being bailed-out too, if necessary. This assistance mechanism can be an *ad hoc* solution or a predefined arrangement. The European Union attitude towards the crisis had different phases and different solutions, which are a good illustration of the different options. The first Greek bail-out (2010) - when the financial envelope was gathered, in part, with bilateral loans from other countries – was an *ad hoc* solution but, after that, new instruments were created and now exists a permanent European bailout mechanism – the European Stability Mechanism (ESM).

The flipside of the creation of these new mechanisms were further rules and economic conditionality (namely in terms of fiscal targets and structural reforms, inspired in International Monetary Fund programs), which are a normal feature in this kind of institutional arrangements. Those who share the risk are interested not only in the 'selfish' view of paying a cost that, indirectly, could be their own cost (by contagion) but also because they want to have a word to say about the policies of their neighbor countries. Fiscal compact treaty in the euro area is an example of such requirements.

In this paper, we will try to evaluate some of these questions within an agent-based model framework for heterogeneous countries: crisis contagion (or serial defaults), bail-out mechanisms and monetary policy role. The idea is to analyze the problem in abstract terms for different scenarios and draw some conclusions that may be useful in understanding some of the problems in Eurozone.

The starting point is the model presented by Jean Tirole (Tirole, 2015) in which he attempts to determine the optimal strategies for countries and optimal contract design that maximizes utility in default scenarios for different frameworks. Our goal is to simulate sovereign default contagion in a regional context where one country default has implications for its neighbors, where it is possible to establish *ex-ante* agreements for risk sharing and where the central bank has some tools to deal with default risk. An environment similar to the one faced by Eurozone.

4.2 Literature review

There are, at least, four strands of literature related to our topic: agent-based models for financial contagion, financial contagion itself (not computational), sudden stops literature and the role of monetary policy preventing sovereign default.

Regarding the first subject, it is very easy to find examples in the literature of papers using computational economics to deal with financial contagion, namely since 2007/2008 financial crisis. However, it is very difficult to find examples of agent-based models or other computational tools dealing directly with sovereign default. The most frequent subjects are credit flows and financial system itself.

Steinbacher *et al* (2013), for example, used a network system to assess the credit contagion channel in financial markets and concluded that the effects are non-linear and shocks transmission depends heavily on the financial system structure and on the functioning of the interbank market. Zedda (2014) used simulations to test not only the 'pure' financial contagion but also the consequences for public finances and real economy, which he calls the "side effects" of systemic crises. Galliani and Zedda (2015) also look to the "vicious circle" between banks and public debt. They conclude that this is a "real threat" and that the shock tends to disappear only if the bank collapses are not severe or if the system is strong enough to absorb the impact.

Klimek, Poledna, Farmer and Thurner (2015) used an agent-based model to simulate the bail-out and/or bail-in of distressed financial institutions in an environment where governments can choose between three alternatives: closing the bank, bail-in it or bail-out it. Simulations were performed in CRISIS macromodel.

Bookstaber, Paddrik, and Tivnan (2014) simulate a fire sales scenario in order to understand the mechanism behind stop phenomena, for example. Bookstaber (2012) tested financial vulnerabilities.

Caporale, Serguieva and Wu (2009) used ABM models to test different strategies in a financial crisis scenario, in order to define parameter values and characteristics useful for early detection of financial contagion or powerful financial crisis. Caporale, Serguieva and Wu (2008) had already used an ABM model for simulating financial contagion.

Lengnick and Wohltmann (2013) present a synthesis between ABM models and new-keynesian macroeconomics. In particular, they use a NK model mixed with financial markets ABM model features, namely the fundamentalist-chartist model.

There are, in recent years, some examples in the literature about sovereign debt and fiscal policy simulation using ABM. Raberto, Teglio and Cincotti (2011) use Eurace simulator to understand linkages between financial sector and economic performance. Thurner (2011) presents an extensive review about the use of ABM to evaluate and assess risks related with nation-level leverage and economic indicators. Pick and Anthony (2006) applied a simulation model to assess UK debt strategy. Gande and Parsley

(2005) analyses the rating downgrades contagion on neighbor countries and it concludes that it is highly asymmetric: downgrades have negative impact; upgrades don't have any.

The second stream of research targets financial contagion itself, away from simulation or ABM models. For instance, Allen and Gale (2000) presents one of the classic approaches to financial contagion through liquidity preference shocks. This kind of turbulence can be easily used to interpret sovereign debt crisis in Eurozone and the flight to quality phenomena experienced by countries like Portugal, Greece or Ireland in the run up to their respective bail-outs.

On a more empirical basis, Kaminsky and Reinhart (2000) look to trade and financial links as contagion channels based on the data for 80 currency crisis between 1970 and 1998. In order to assess the role of international lending, cross-market hedging and trade, they find that contagion is a non-linear process and contagion channels are not always the same in different crisis.

Dornbusch, Park and Claessens (2000) present a taxonomy of the financial crisis contagion based on specific examples, literature analysis and empirical results.

Full understanding of traditional propagation mechanisms are also useful to anticipate crisis contagion, as done by Schimmelpfenning, Roubini and Manasse (2003). The three authors proposed a logit model and a binary recursive tree that are effective early warning mechanisms in, respectively, 74% and 89% of the crisis.

Some papers, like Lizarazo (2009), use a theoretical framework to evaluate crisis contagion, namely a DSGE model of default risk to identify endogenous foundations of the contagion. Theoretical results were in line with empirical evidence of Argentina-Uruguay contagion and suggested that: a) sovereign spreads and capital flows are correlated; b) economic fundamentals affect sovereign spreads and capital flows; and c) financing conditions in one economy are less favorable after other countries defaulted.

Constâncio (2012) and Kalbaska and Gatkowski (2012) used credit-default swap (CDS) spreads in Eurozone to detect financial contagion. Constâncio argue that contagion played a more important role than fundamentals in sovereign debt crisis in EMU. Kalbaska and Gatkowski concluded that contagion exists but it is different among countries and that Portugal is one of the most fragile economies.

Mink and De Haan (2013) analyzed bank returns across Europe in response to news about Greece in 2010 and found that both news about Greek economy or Greek bailout had an impact. Beirne and Frarzscher (2013) found that, more than a contagion *per se*, sovereign debt crisis in Europe propagated across continent because economic agents – investors, in particular – decided to consider fundamentals in their decisions.

The third branch of research useful when studying EMU debt crisis – or a sovereign debt crisis - is the sudden stops literature. In some aspects, flight to quality and external reluctance to maintain investments in some specific countries is clearly a sudden stop phenomena. Merler and Pisani-Ferry (2012) consider that the massive capital outflows of some Eurozone countries can be qualified as sudden stops and that demonstrates that balance of payment crisis are still possible in the context of a monetary union.

Cavallo and Frankel (2008) used a gravity model for Latin America and find that openness is associated to less sudden stop and currency crashes risks. This result was not applied directly to European countries but should be considered in any future institutional revisions that might take place in EMU.

Other paper about sudden stops is Mendonza (2010) which explores the linkages between sudden stops and economic crisis looking to the role of the collateral constraints. It argues that this kind of phenomena has non-linear and asymmetric features.

Argentinian crisis is the object of Calvo, Izquierdo and Talvi (2003) paper where the three authors offer a sudden stop explanation for the peso-dollar peg collapse. Calvo, Izquierdo and Mejia (2004) present an empirical analysis of sudden stop crisis based on a sample of 32 developed and developing countries and concluded that sudden stops with large real exchange rate fluctuations are an emerging market phenomena and seem to come in bunches - grouping countries that are apparently different. Calvo (1998) presents the “simple economics of sudden stops”.

Finally, the last strand of literature is related to the role of monetary policy dealing with sovereign default. For example, Schabert (2010) “examines equilibrium determination under different monetary policy regimes when the government might default on its debt" in a cash-in-advance theoretical model. Schabert and Van Wijnbergen (2014) looked at the linkages between sovereign default and inflation targeting regimes, concluding that "if fears of debt default are positively correlated with the debt service burden, unstable cycles are a possibility" and monetary policy, under this kind of regime, "will destabilize the economy".

Daniel and Shiamptanis (2012) simulate fiscal risk within a monetary union, with an application to the EMU scenario in 2009, to analyze trade-offs between monetary policy and fiscal options such as a default or policy shift.

4.3 The model

Our model is adapted from Jean Tirole´s model with two main structural differences: Tirole´s model is a game theory model to define optimal behaviors while our model intends to simulate contagion and not optimal strategies; Tirole´s model has only two countries, two periods and considers two different cases and our model have a large number of periods and different countries[27].

There are two another important departures from Tirole´s model related with the governments decisions: discretionary default is not possible and countries only default when they don´t have enough income to pay; the penalty of a default is being out of the markets until the 'end of times' without the possibility of borrowing.

[27] The general dynamic of Tirole´s model is the following. In the scenario without *ex-ante* agreements (*laissez-faire*), country A (the Agent) decides how much to borrow (b) in period 1. In period 2, it has an income y and decides to pay an amount d of the loan. If this payment d is not the full repayment, the country has a penalty (or cost) c which, in turn, indirectly affects the other country P (the Principal) in an amount Rc (R is a value between 0 and1). Income y depends on the state of nature and it is only observable by the country itself. In this general framework, the two countries can make bilateral agreements where both can have utility gains. In *ex-ante* agreements, country A, in exchange for a transfer τ, commits to a contract that limits its borrowing level and determines a penalty for the case of partial or total default. In *ex-post* agreements, after the state of nature materializes in period 2, country P transfers a value to assure the loan repayment. For more details, see Tirole (2015).

4.3.1 Countries

We considered a world with heterogeneous countries in terms of dimension (Small and Large), propensity to borrow (Spenders and Savers) and risk-aversion levels (High-risk and Low-risk averse). These features are mixed so we can have all the different combinations of these three dimensions, randomly generated.

Formally, we have N countries in our model:

- S small countries and $(N - S)$ large countries
- X Spenders and $(N - X)$ Savers
- R have High-risk aversion and $(N - R)$ have Low risk aversion

4.3.2 Income and borrowing

In each period, countries have a random income following a uniform distribution in predefined intervals[28]. Small countries have income y_S:

$$y_S = [y_S^{min}, y_S^{max}] \tag{1}$$

And Large countries have income y_L:

$$y_L = [y_L^{min}, y_L^{max}] \tag{2}$$

To smooth their spending, countries can borrow a certain amount that should be fully payed (with interest) in the next period. Formally, each country i borrows $b_{i,t}$ in period t and pay $d_{i,t+1}$ in period t_{i+1} (it is assumed that each credit has a maturity of only one period).

Borrowing serves to compensate income volatility within the intervals. Spenders compensate a γ part of the difference to maximum income while Savers compensate only $\emptyset$. They have to consider also an interest that increases their payment in next period with the rate $i_{i,t}$ that will be

[28] In the original Tirole´s model, income was y with probability α and 0 with probability (1-α).

presented in further detail in section 3.3. This means that their borrowing limit will consider the interest.

$b_{i,t}^{X}$ and $b_{i,t}^{S}$ are, respectively, Spenders and Savers borrowing in period t:

$$b_{i,t}^{X} = \frac{\gamma\,(y^{max} - y_{i,t})}{(1 + i_{i,t})} \tag{3}$$

$$b_{i,t}^{S} = \frac{\emptyset(y^{max} - y_{i,t})}{(1 + i_{i,t})} \tag{4}$$

with $0 < \emptyset, \gamma < 1$ and $\gamma > \emptyset$.

4.3.3 Paying, default and disposable income

In every period t, countries have a random income, pay their debts (the borrowing from the previous period with due interest) and decide how much to borrow again. Each country pays $d_{i,t}$ in period t that is given by:

$$d_{i,t} = b_{i,t-1}\,(1 + i_{i,t-1}) \tag{5}$$

where is $b_{i,t-1}$ is the previous period borrowing and $i_{i,t-1}$ is the country i interest rate in the previous period.

This country-specific interest rate $i_{i,t}$ can be written has:

$$i_{i,t} = i^{*} + s_{i,t} \tag{6}$$

where i^{*} is the central bank main interest rate (equal for all market participants) and $s_{i,t}$ is the country i spread in period t. This spread depends on the debt-to-income ratio of the country and on the existence of *ex-ante* agreements. Countries with agreements pay no spread, the others have a spread positively related with their debt-to-disposable income ratio given by:

$$D_{i,t} = \frac{d_{i,t-1}}{YD_{i,t}} \tag{7}$$

There are five different spreads for different debt ratios thresholds:

$$s_{i,t} = \begin{cases} 1, & D_{i,t} < 0.2 \\ 2, & 0.2 \leq D_{i,t} < 0.4 \\ 3, & 0.4 \leq D_{i,t} < 0.6 \\ 4, & 0.6 \leq D_{i,t} < 0,8 \\ 5, & D_{i,t} \geq 0,8 \end{cases} \tag{8}$$

As we said earlier, country pays all (if its income is enough) or nothing. Strategic (total or partial) default is not possible. When the 'neighbor' don´t pay, the country has a Rc cost, with different R based on the linear distance to the defaulting country. In the simplest form, used in our simulation, we considers $R = 1$ for all countries.

To evaluate individual (and aggregate) economic performance, we compute individual country disposable income *per period* ($YD_{i,t}$) that is simply the difference between receipts (income, borrowing and agreement transfers) and spending (previous debt and financial impact of defaults). It is given by the following expression for country i in period t:

$$YD_{i,t} = y_{i,t} + b_{i,t} - d_{i,t} + \tau - \frac{D_{t-1}.c}{(N - D_{t-1})} \tag{9}$$

where D_{t-1} is the number countries defaulting in period t-1, τ is the transfer for countries with agreements and the other variables are the same used previously.

4.3.4 *Ex-ante* agreements

In each period, countries decides about entering in agreements. These agreements protects them against default: if, by any chance, this happens in financial terms (their income is not enough) they are implicitly bailed-out and will keep on the markets borrowing normally. The countries receive also a transfer – a kind of fiscal transfer for a country within a fiscal union or something similar – but have commit to rules related to spending behavior.

Ex-ante agreements transform Spenders in Savers, which means that default risk decrease in exchange of a transfer τ. *Ex-ante* agreements with Savers transform them in Ultra-Savers (with a lower borrowing parameter $\mu < \emptyset$).

The decision to enter in an agreement depends on the risk aversion level. Higher risk aversion countries tends to be more afraid of a default scenario so they enter more rapidly in agreements to protect themselves when this scenario seems plausible. In concrete terms, countries enter in *ex-ante* agreements when their income falls below a certain threshold.

Low risk-averse countries decide to enter in agreements when their income is lower than the midpoint of the income interval for the first time, while High risk-averse countries have a higher threshold (3/4 of the interval).

The cost of agreements, i.e. the transfers made to participant countries, are equally divided by all the remaining countries. In our simpler form, this cost is not imputed to any country, but this doesn´t change the general conclusions.

4.3.5 Monetary policy

In this model, monetary policy as a very important role shaping the interest rates faced by the countries when borrowing. This feature has a high adherence to reality where, in fact, central banks can have a huge influence in market interest rates in different segments and/or maturities. In this case, monetary policy affects interest through two different channels. First, central bank sets the main interest rate i^* that all countries pay equally. Second, central bank can engage in quantitative-easing-style policies that have a horizontal impact in all interest rates, meaning that countries tends to pay similar interest rates, i.e. equal spreads (or no spreads at all). For the sake of simplicity, we will consider that with QE spreads are all zero.

The objective of using this monetary policy instruments is to analyze the impact of low interest rate and QE policies in default rates. In the particular case of the QE-style policy, some moral hazard problems may arise from the fact that without spreads, countries don´t have market pressure to limit their borrowing. In this model we don´t have any cycle variable (an output gap, for instance) nor inflation that could be targeted by the central bank, so the different interest rate levels and QE usage are exogenous variables.

4.4 Simulation and results

We considered 40 periods and 20 different countries for simulation purposes. It is an arbitrary choice that can be, of course, changed. But the underlying rational is to have a number of countries close to the real number of EMU countries, during a 10 year period similar to this 10 year crisis since 2007/2008. Because we are thinking in terms of quarterly decisions, which are enough time to governments to have decisions on spending and borrowing and to markers react, these 10 years corresponds to 40 quarters. Our main goal is simply to generate enough decisions and interactions to artificially create an environment of sovereign default contagion for different scenarios that allow us to withdraw some useful conclusions.

In each of these 40 periods, countries have an income $y_{i,t}$ randomly generated in two different intervals [10,30] and [30,70], respectively for Small and Large countries. Countries with individual income lower than the sum of previous period debt and contagion cost of other countries default enter in default. This means that they will be out of market until the last period, living only with their 'natural income' randomly generated.

However, countries with an agreement – that imposes a limit on their borrowing – may default in financial terms (their income being lower than their financial needs) but stay in the market normally. For the sake of simplicity, there are no creditors of this debt, which is the same of saying that there is no financial system supplying funds to the countries. This is a research avenue for future developments of this paper. In this stage, the only financial consequence of a default for neighbor countries is the impact c of each default and this impact is the same for Small and Large countries. This means that, for a constant contagion cost, Small countries suffer most.

After that, the countries that are not in default or don´t have previous agreement, decides if they want to enter an agreement – exchanging a transfer τ for a more frugal behavior. Only then, the country will decide how much to borrow depending on their specific spending stance.

In simple terms, the process is divided in four basic steps:

1) Countries have an income $y_{i,t}$
2) Income is compared with previous period borrowing ($b_{i,t-1}$) with interest, added to default contagion impact (measured by the sum of c factor divided by the numbers of countries not in default). If income is enough, country pays. Otherwise, the country will default which means that, if it has no prior agreement, it will be out of the markets forever.
3) Countries that are not in default and have no previous agreement decide if they want to adopt one. This decision depends on their individual risk aversion level – that determines a specific threshold to trigger a decision – and on their income.
4) Countries decide how much to borrow and this decision depends on their propensity to borrow. Borrowing is used to compensate income volatility. In practice, they borrow an amount depending on the distance between current income and maximum possible income. Spenders have a larger borrowing parameter while Savers have a lower parameter. Countries with ex-ante agreements change their behavior: Spenders become Savers and Savers become Ultra-savers.

4.4.1 Scenarios and parameters

Simulation was performed for six different scenarios. All have the same number of periods (40) and countries (20) but different partitions in terms of Small/Large countries, Spenders/Savers, High/Low risk averse countries and monetary policy parameters. *Ex-ante* agreement transfer (τ) and contagion cost (c) are set, respectively, to 10 and 5. Borrowing parameters are set to 1, 0.7 and 0.4, respectively for Spenders, Savers and Ultra-savers.

Debt is randomly initialized for $t = 0$ without considering interest capitalization in the decision because there is no debt-to-income ratio prior to this debt. From $t = 0$ to $t = P$, each borrowing amount will pay interest base on the rate determined for that same period but the decision is based on previous period interest rate – that serves as proxy because countries only know how much they will when effectively tap the markets.

Table 4.1 summarizes the six scenarios:

<table>
<tr><th>SCENARIO</th><th>P</th><th>N</th><th>L</th><th>X</th><th>R</th><th>γ</th><th>ϕ</th><th>μ</th><th>τ</th><th>c</th><th>i^*</th><th>QE</th></tr>
<tr><td>Baseline</td><td rowspan="6">40</td><td rowspan="6">20</td><td>5</td><td>10</td><td>5</td><td rowspan="6">1</td><td rowspan="6">0.9</td><td rowspan="6">0.5</td><td rowspan="6">10</td><td rowspan="6">100</td><td>0.2</td><td>0</td></tr>
<tr><td>2</td><td>2</td><td>10</td><td>5</td><td>0.2</td><td>0</td></tr>
<tr><td>3</td><td>5</td><td>5</td><td>5</td><td>0.2</td><td>0</td></tr>
<tr><td>4</td><td>5</td><td>10</td><td>10</td><td>0.2</td><td>0</td></tr>
<tr><td>5</td><td>5</td><td>10</td><td>5</td><td>0</td><td>0</td></tr>
<tr><td>6</td><td>5</td><td>10</td><td>5</td><td>0</td><td>1</td></tr>
</table>

Note: Scenario 2 have less large countries (2); Scenario 3 have less spenders (5); Scenario 4 have higher risk aversion (10); Scenario 5 has 0% main interest rate without QE; Scenario 6 has 0% main interest rate with QE

Table 4.1 – Simulation scenarios

Baseline scenario has 5 Large countries, 10 Spenders and 5 High-risk averse countries. Scenarios 2 to 4 are variations among these three dimensions: Scenario 2 depicts a world with less Large countries (2 instead of 5); Scenario 3 has less Spenders (5 instead of 10) and Scenario 4 has more High risk aversion countries (10 instead of 5). The last two scenarios – 5 and 6 – looks closely to monetary policy impact: zero interest rate without (5) and with quantitative easing (6).

4.4.2 Baseline results

Several conclusions can be drawn regarding sovereign default propagation and income volatility in a world with different countries and different behaviors towards borrowing. Baseline scenario results provide very interesting and revealing results. Next we will provide some examples of a typical run of the program.

First, average income is highly volatile, which is a direct result of its own nature – it is randomly generated – and to the fact that countries change their borrowing attitudes after defaulting or after an agreement. This can be easily seen in Figure 4.1.

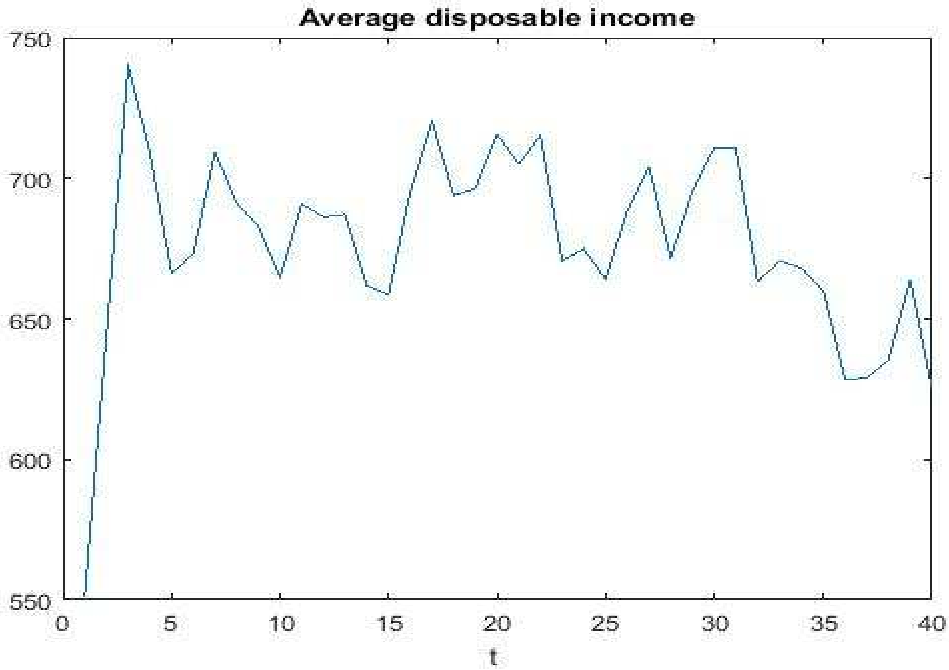

Figure 4.1 – Baseline: average income

Second, Large countries have larger average incomes. That is normal considering that, by definition, large countries have always larger incomes, as we can see in Figure 4.2.

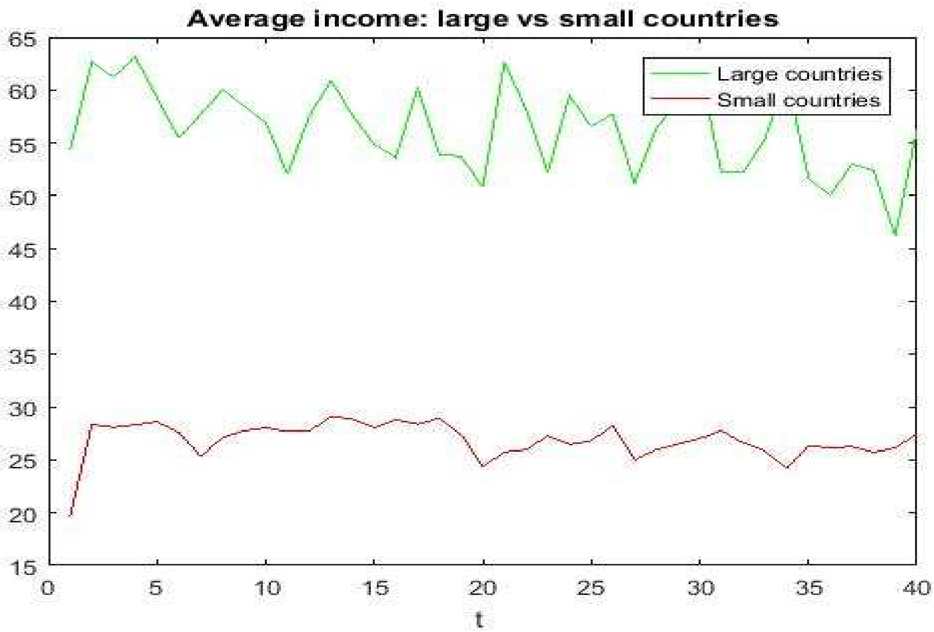

Figure 4.2 – Baseline: large versus small countries

Third, Savers tend to have higher disposable incomes. It is probably due to the fact that Savers have lower default rates and can stay in the market borrowing normally. On the contrary, Spenders tend to have earlier defaults with consequences for their future income. This pattern can be easily recognized in Figure 4.3.

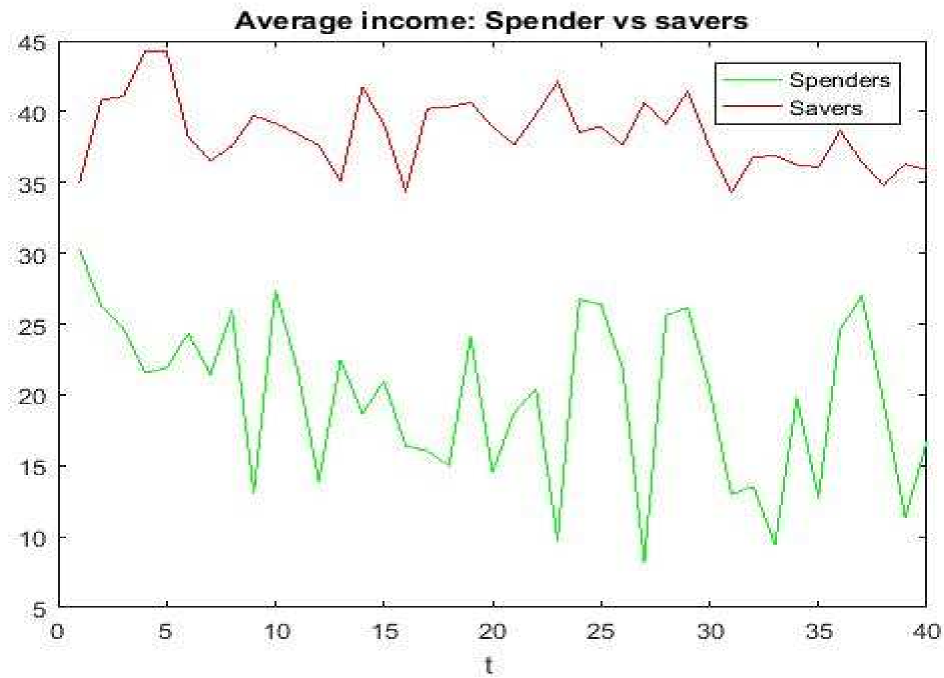

Figure 4.3 – Baseline: spenders versus savers

In this run, 5 countries defaulted during the 40 periods time span, as we can see in Figure 4.4.

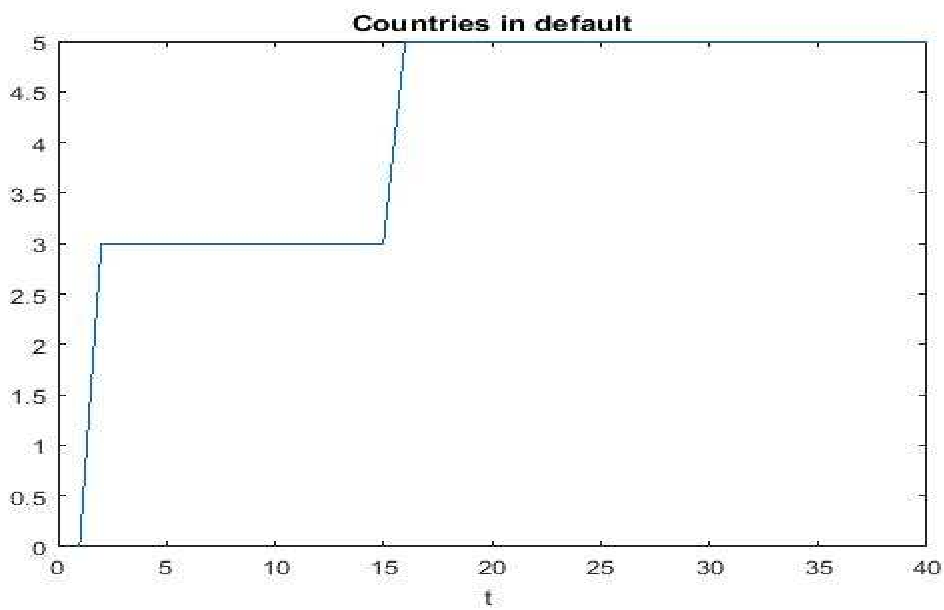

Figure 4.4 – Baseline: number of defaults

4.4.3 Scenario comparison

Economic performance of Scenario 3 tend to be higher than all the others. It has a simple explanation: less Spenders are associate with lower default levels and contagion, so more countries stay in the market borrowing normally. This means that disposable income (total and average) tend to be higher than in other scenarios.

All the total and average disposable incomes for the four scenarios are depicted in Figures 4.5 and 4.6.

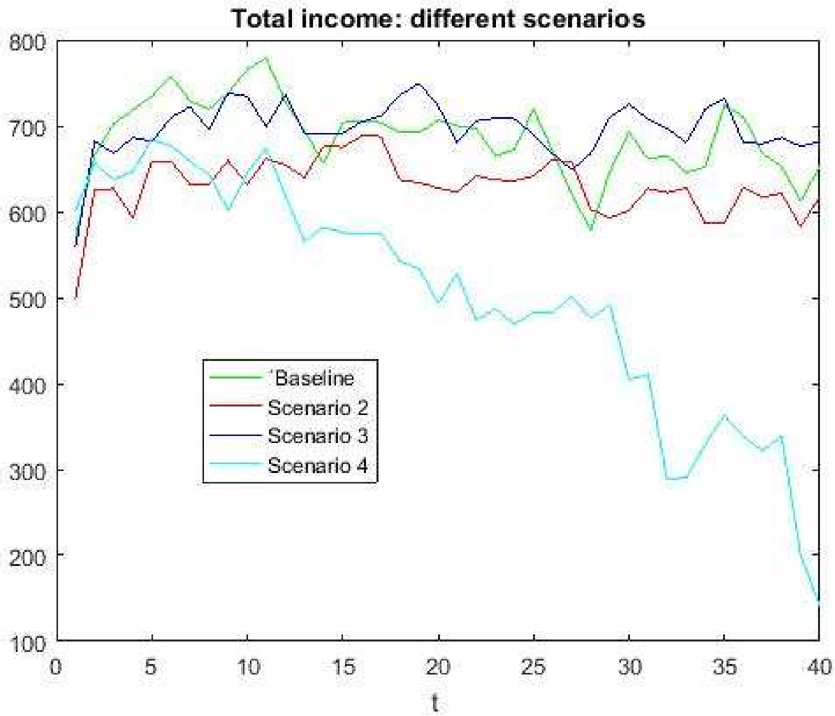

Figure 4.5 – Scenarios total income performance

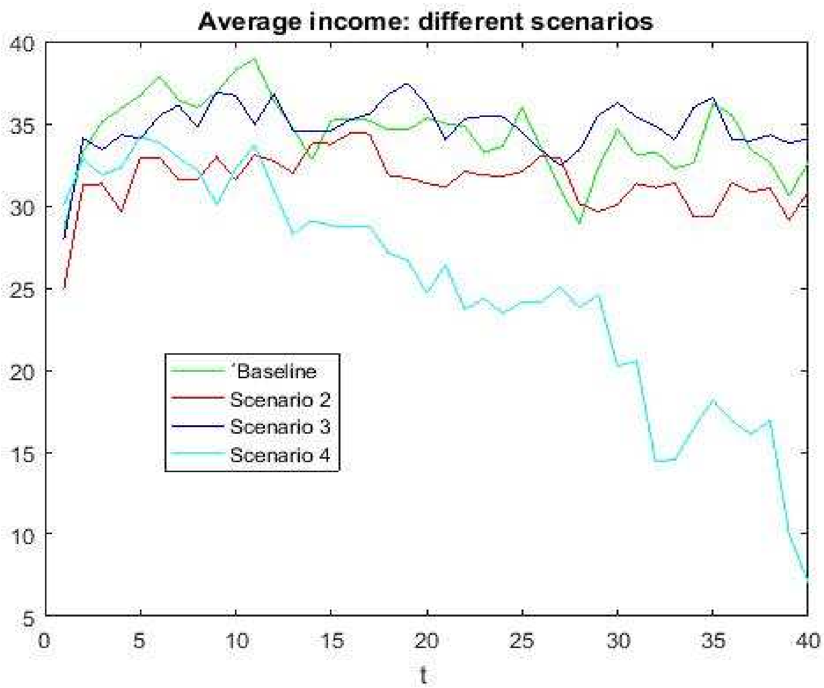

Figure 4.6 – Scenarios average income performance

Looking to the number of defaults, the conclusions are roughly the same. Scenario 3 has the best performance (lowest number of defaults) and Scenario 4 has the worst. This can be seen in the typical run of Figure 4.7 but it is also a regular result. If we consider the average results after 100 runs, Baseline has 7.25 defaults, while Scenarios 2,3 and 4 have, respectively, 10.8, 3.3 and 7.21 defaults. The results are presented in table 4.2.

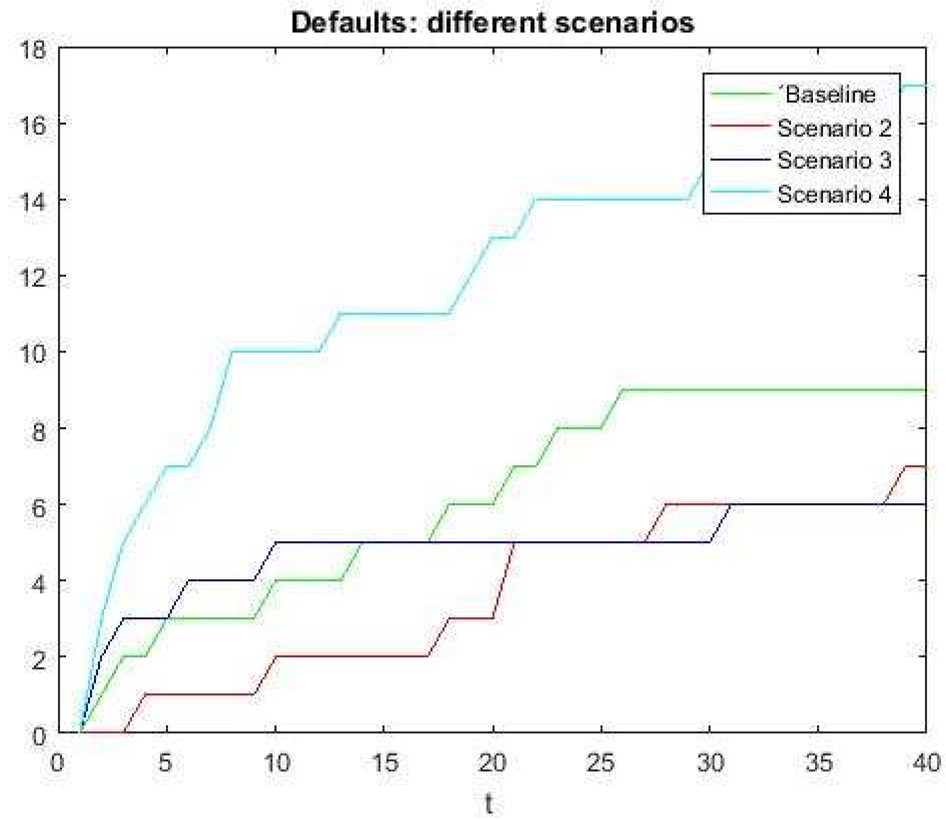

Figure 4.7 – Number of defaults

Baseline	Scenario 2	Scenario 3	Scenario 4
7.25	10.80	3.30	7.21

Table 4.2 – Average number of defaults in different scenarios for countries

4.4.4 Monetary policy impact

When we compare Baseline performance against scenarios with monetary stimulus - scenario 5 with 0% interest rate and scenario 6 with 0% interest rate and QE – the results are also revealing. Baseline has a poorer performance in terms of total and average income and defaults.

Monetary policy, as we said earlier, has a double impact in economic performance and default rates: it lowers the general interest rate with the main interest rate and can have a zero spread effect with QE. This means that, when we compare the Baseline Scenario with the two scenarios with two different degrees of monetary stimulus, the impact is easily recognized.

Figures 4.8 and 4.9 represent this effect comparing total and average incomes of the three scenarios.

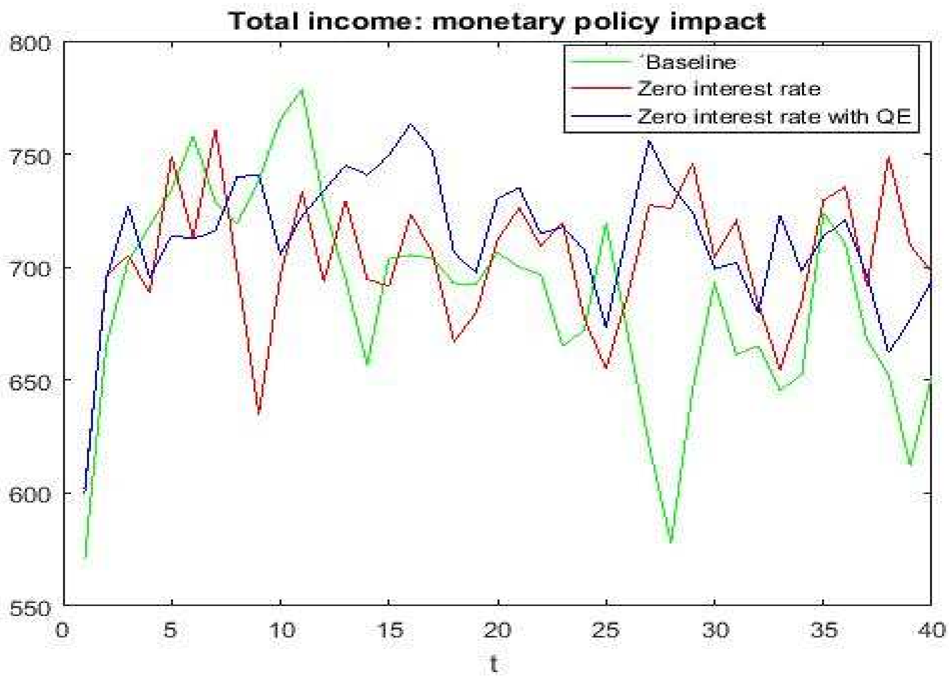

Figure 4.8 – Total income for monetary policy scenarios

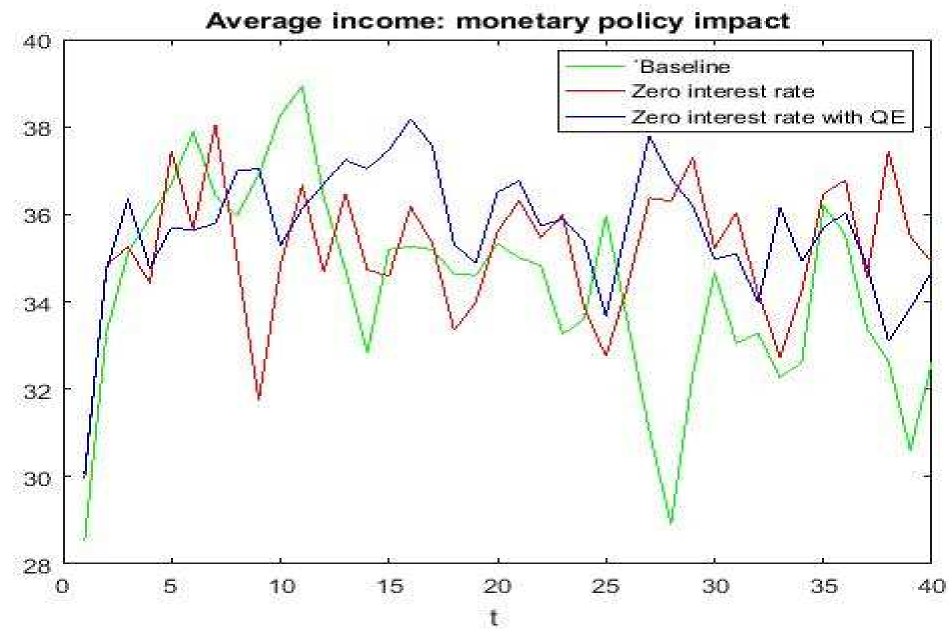

Figure 4.9 – Average income for monetary policy scenarios

Figure 4.10 depicts a typical run to compare default rates that, also, reveal the impact of monetary policy in preventing countries financial collapses. In fact, after 100 runs, Baseline has, on average, a higher number of defaults as represented in table 3: 7.25 against 7.01 and 6.97. Scenarios 5 and 6 have almost identical performance in terms of default, with (surprisingly) a little advantage to scenario with QE.

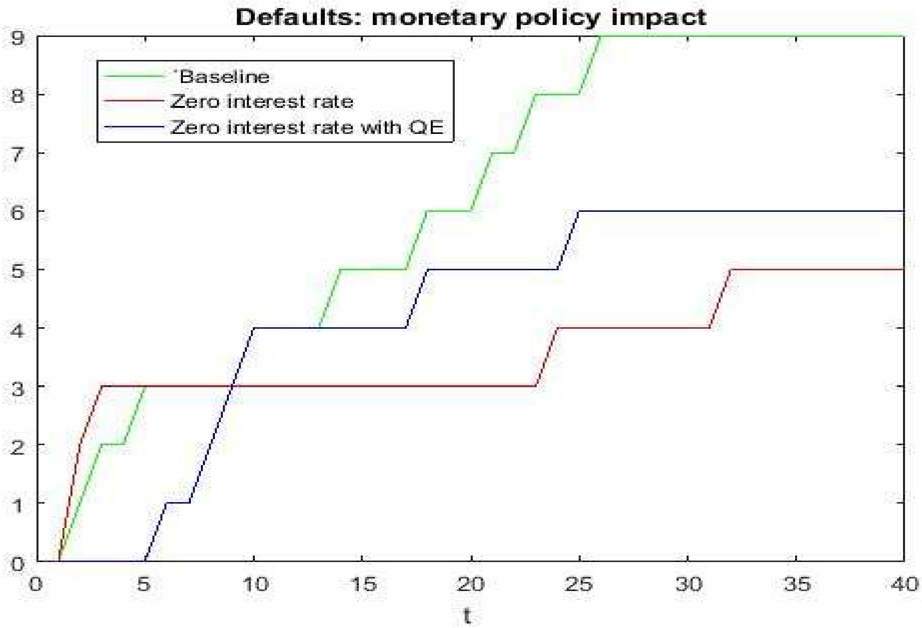

Figure 4.10 – Number of defaults

Baseline	Scenario 5	Scenario 6
7.25	7.01	6.97

Table 4.3 – Monetary policy impact in the number of defaults

4.5 Concluding remarks

In this paper we simulate sovereign default contagion in an environment where countries have random incomes and different borrowing behaviors and risk aversion levels. Countries can decide to enter in *ex-ante* agreements with other countries in order limit their borrowing in exchange for a transfer and a hedge against default. We tested also the impact of monetary policy options, namely zero interest rates and QE-style instruments.

The model was based on Jean Tirole´s (Tirole, 2015) framework, with some extensions. In particular, we considered a larger number of countries and periods as well as modifications in the borrowing procedure and in the contagion process.

The main conclusions are:

a) higher risk aversion is associated with lower default levels but also with lower disposable incomes
b) countries with higher propensity to borrow (Spenders) tend to have lower average disposable incomes
c) default contagion is a very rapid process and can affect all the countries in a small number of periods
d) monetary policy can have a concrete effect preventing sovereign defaults through zero-interest rate policies but also through non-conventional measures
e) QE instruments increases the interest rate policy impact but can have moral hazard consequences that is a topic beyond the scope of this paper.

This work is only a first step to simulate sovereign default contagion. Several improvements can be made in the future along two main roads of research. First, the model specification itself. For example: improving the contagion process to consider bilateral distance between countries or even more complex bilateral relations based on economic ties; consider different contagion effect for large and small countries; consider time-depending risk aversion or other time-depending variables; introducing cycle variables (output gap, for example) and monetary policy rules.

The second road of improvement is related to the perimeter of the model – introducing a banking sector as the creditor of the countries, for instance - and with the introduction of external shocks – to simulate financial crises periods for example.

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4.7 ODD protocol

4.7.1 Overview

Purpose: The objective of this model is to evaluate sovereign debt crisis contagion (or serial defaults), bail-out mechanisms and monetary policy role in an environment with heterogeneous countries and different policy options. The countries have different borrowing propensities, different risk aversion preferences and face uncertainty about their future income.

State variables and scales:

The agents have three individual characteristics: dimension (small and large); propensity to borrow (spenders and savers) and risk-aversion levels (high-risk and low-risk). Depending on the dynamics, some of the countries can change their behavior towards borrowing from spenders to savers and savers to ultra-savers. The countries income in each period is randomly attributed in an interval depending on their dimension. They can borrow to smooth their disposable income and pay an interest rate that has two components: idiosyncratic component and a general component.

The agent´s parameters are:

- γ , $\emptyset$ and μ which represents the borrowing parameter, respectively, for spenders, savers and ultra-savers
- $s_{i,t}$ is the country- specific interest rate spread

The state variables are:

- i^* is the central bank rate

Process overview and scheduling: The process is divided in four basic steps repeated in each t period:

- Countries have an income $y_{i,t}$
- Income is compared with previous period borrowing ($b_{i,t-1}$) with interest and default contagion impact. If the income is enough, country pays, otherwise it will default. If the country has no prior agreement, it will be out of the markets forever.
- Countries that are not in default and have no previous agreement decide if they want to adopt one, depending on their risk aversion level and income.
- Countries decide how much to borrow. Their borrowing level depends on the distance between current income and maximum possible income.

4.7.2 Design concepts

Emergence: From individual decisions and behaviors, the system provides the emergence of some global patterns related with sovereign default contagion. In some scenarios, contagion phenomena can be a very fast and destructive process. At the same time, the monetary policy options can give some macro trend homogeneity to the system.

Adaptation: The agents (countries) have to deal with different borrowing decisions in every period but the most relevant adaptation feature of the system is the possible change in the countries behavior depending on their risk aversion level and the evolution of their income.

Fitness: System performance is determined at a global level based on income growth and the number defaults. At a local (individual) level, countries are compared in terms of their individual characteristics, namely dimension, propensity to borrow and risk aversion.

Prediction: Agents don´t have any special ability in terms of anticipation or prediction of the future path of the system. They have, however, some simple rules to determine how much they borrow, how they decide about entering agreements or how they change their behavior.

Interaction: Countries interact directly because one country default has consequences on its neighbors. And they interact with the system because it is the general state of the world that determines their income and because monetary policy stance have a crucial role in interest rate supported by the countries.

Stochasticity: Income is the most important stochastic variable of the system and it is its main driver. But there are also other stochastic elements related with the creation of the system itself when countries characteristics are randomly generated.

Collectives: Model have different group of agents based on their characteristics, which allow us to draw several conclusions when we compare their relative performance, namely Large versus Small, Spenders versus Savers or default versus non-default.

Observation: Results are presented in terms of accumulated growth, default numbers and other variables. We present also figures comparing the performance of different groups.

4.7.3 Details

Initialization: Countries characteristics and other variables are randomly initialized in $t = 0$. The initial debt level don´t consider any interest because there is no prior debt-to-income ratio.

Input: Simulation is performed for six different scenarios. All the scenarios consider 40 periods and 20 countries, but adopt different partitions in terms of Small/Large countries, Spenders/Savers, High/Low risk averse countries and monetary policy parameters. *Ex-ante* agreement transfer (τ) and contagion cost (c) are set, respectively, to 10 and 5. Borrowing parameters are set to 1, 0.7 and 0.4, respectively for Spenders, Savers and Ultra-savers.

4.8 Appendix: Matlab

```
function [result]=default(P,N,L,X,R,g,s,u,t,c,i,Q)
% P=periods
% N=countries
% L=large countries
% X=spenders
% R=high risk aversion countries
% g=spenders parameter
% s=savers parameter
% u=ultrasavers parameter
% t=bilateral agreement transfer
% c=default contagion parameter
% i=central bank rate
% Q=central bank QE (0 for QE and spreads=0)
% baseline run result=defaultoutubro2017(100,20,5,10,5,1.08,0.5,0.1,10,100,2,1);


% Countries initiation
M=zeros(N,14);  % 1 large,2 spenders, 3 high risk aversion, 4 current income, 5 previous period debt,6 current debt, 7 current payment, 8 default, 9 agreement, 10 transfer, 11 contagion, 12 disposable income, 13 debt ratio, 14 spread
Mrand=randperm(N);
```

```
M(Mrand(1:L),1)=ones(L,1); % large countries
Mrand=randperm(N);
M(Mrand(1:X),2)=ones(X,1); % spenders
Mrand=randperm(N);
M(Mrand(1:R),3)=ones(R,1); % high risk aversion countries
% Other variables initiation; small countries income (10-30);large countries income (30-70);

FS=find(M(:,1)==0);
M(FS,4)=10+(30-10)*rand(size(FS)); % small countries income

FL=find(M(:,1)==1);
M(FL,4)=30+(70-30)*rand(size(FL)); % large countries income

FSX=find(M(:,1)==0 & M(:,2)==1);
M(FSX,5)=g*(30-M(FSX,4)); % initial previous debt for small spenders
FSS=find(M(:,1)==0 & M(:,2)==0);
M(FSS,5)=s*(30-M(FSS,4)); % initial previous debt for small savers
FLX=find(M(:,1)==1 & M(:,2)==1);
M(FLX,5)=g*(70-M(FLX,4)); % initial previous debt for large spenders

FLS=find(M(:,1)==1 & M(:,2)==0);
M(FLS,5)=s*(70-M(FLS,4)); % initial previous debt for large savers
M(:,12)=M(:,4)+M(:,6)-M(:,7)+M(:,10)-M(:,11); % initial disposable income

M(:,13)=M(:,5)./M(:,12); % initial debt ratio (previous period debt/disposable income)
FDH1=find(M(:,13)<=0.2);
M(FDH1,14)=1*Q; % spread countries debt<20%
FDH2=find(M(:,13)>0.2& M(:,13)<=0.4);
M(FDH2,14)=2*Q; % spread countries debt between 20% and 40%
FDH3=find(M(:,13)>0.4& M(:,13)<=0.6);
M(FDH3,14)=3*Q; % spread countries debt between 40% and 60%
FDH4=find(M(:,13)>0.6 & M(:,13)<=0.8);
M(FDH4,14)=4*Q; % spread countries debt between 60% and 80%
FDH5=find(M(:,13)>0.8);
M(FDH5,14)=5*Q; % spread countries debt>80%
```

```
% cicle
for z=1:P
    % INCOME COMPUTATION
    FS=find(M(:,1)==0);
    M(FS,4)=10+(30-10)*rand(size(FS)); % small countries income
    FL=find(M(:,1)==1);
    M(FL,4)=30+(70-30)*rand(size(FL)); % large countries income

    % PAYMENT OR DEFAULT
    FF=find(M(:,8)==1); % find countries in default
    D(z)=size(FF,1); % number of default countries

    FND=find(M(:,4)-D(z)*c/(N-D(z))>=M(:,5).*(1+i/100+M(:,14)/100-
M(:,14)/100.*M(:,9))); % countries able to pay
    M(FND,7)=M(FND,5).*(1+i/100+M(FND,14)/100-
M(FND,14)./100.*M(FND,9)); % payment for countries with enough income
    FD=find(M(:,4)-D(z)*c/(N-D(z))<M(:,5).*(1+i/100+M(:,14)/100-
M(:,14)/100.*M(:,9)));
    M(FD,8)=1; % default for countries without enough income

    % AGREEMENT DECISION

    FLA1=find(M(:,1)==1 & M(:,2)==1 & M(:,3)==1 & M(:,4)<60 & M(:,8)==0
& M(:,9)==0);  % find large, spenders, high risk aversion countries
below threshold
    M(FLA1,9)=1;% countries with agreement
    M(FLA1,10)=t; % agreement transfer
    M(FLA1,2)=0; % spenders become savers
    FLA2=find(M(:,1)==1 & M(:,2)==0 & M(:,3)==1 & M(:,4)<60 & M(:,8)==0
& M(:,9)==0); % find large, savers, high risk aversion countries below
threshold
    M(FLA2,9)=1;  % countries with agreement
    M(FLA2,10)=t; % agreement transfer
    M(FLA2,2)=2; % savers become ultra-savers
    FLA3=find(M(:,1)==1 & M(:,2)==1 & M(:,3)==0 & M(:,4)<50& M(:,8)==0
& M(:,9)==0); % find large, spenders, low risk aversion countries below
threshold
    M(FLA3,9)=1;  % countries with agreement
    M(FLA3,10)=t; % agreement transfer
    M(FLA3,2)=0; % spenders become savers
```

```
    FLA4=find(M(:,1)==1 & M(:,2)==0 & M(:,3)==0 & M(:,4)<50& M(:,8)==0
& M(:,9)==0); % large, savers, low risk aversion countries below
threshold

    M(FLA4,9)=1;  % countries with agreement

    M(FLA4,10)=t; % agreement transfer

    M(FLA4,2)=2; % savers become ultra-savers

    FSA1=find(M(:,1)==0 & M(:,2)==1 & M(:,3)==1 & M(:,4)<25& M(:,8)==0
& M(:,9)==0); % find small, spenders, high risk aversion countries below
threshold

    M(FSA1,9)=1;  % countries with agreement

    M(FSA1,10)=t; % agreement transfer

    M(FSA1,2)=0; % spenders become savers

    FSA2=find(M(:,1)==0 & M(:,2)==0 & M(:,3)==1 & M(:,4)<25& M(:,8)==0
& M(:,9)==0); % find small, savers, high risk aversion countries below
threshold

    M(FSA2,9)=1; % countries with agreement

    M(FSA2,10)=t; % agreement transfer

    M(FSA2,2)=2; % savers become ultra-savers

    FSA3=find(M(:,1)==0 & M(:,2)==1 & M(:,3)==0 & M(:,4)<20& M(:,8)==0
& M(:,9)==0); % find small, spenders, low risk aversion countries below
threshold

    M(FSA3,9)=1; % countries with agreement

    M(FSA3,10)=t; % agreement transfer

    M(FSA3,2)=0; % spenders become savers


    FSA4=find(M(:,1)==0 & M(:,2)==0 & M(:,3)==0 & M(:,4)<20& M(:,8)==0
& M(:,10)==0); % find small, savers, low risk aversion countries below
threshold

    M(FSA4,9)=1; % countries with agreement

    M(FSA4,10)=t; % agreement transfer

    M(FSA4,2)=2; % savers become ultra-savers


 % BORROWING DECISION

    FLB1=find(M(:,1)==1 & M(:,2)==1 & M(:,8)==0); % find large, spenders
without default

    M(FLB1,6)=g*(70-(M(FLB1,4).*(1+i/100+M(FLB1,14)/100-
M(FLB1,14)./100.*M(FLB1,9))));

    FLB2=find(M(:,1)==1 & M(:,2)==1 & M(:,8)==1 & M(:,9)==1);  % find
large, spenders with default and agreement

    M(FLB2,6)=g*(70-(M(FLB2,4).*(1+i/100+M(FLB2,14)/100-
M(FLB2,14)./100.*M(FLB2,9))))

    FLB3=find(M(:,1)==1 & M(:,2)==1 & M(:,8)==1 & M(:,9)==0); % find
large, savers with default and without agreement

    M(FLB3,6)=0;
```

```
FLB4=find(M(:,1)==1 & M(:,2)==0 & M(:,8)==0); % find large, spenders without default

M(FLB4,6)=s*(70-(M(FLB4,4).*(1+i/100+M(FLB4,14)/100-M(FLB4,14)./100.*M(FLB4,9))));

FLB5=find(M(:,1)==1 & M(:,2)==0 & M(:,8)==1 & M(:,9)==1);  % find large, savers with default and agreement

M(FLB5,6)=s*(70-(M(FLB5,4).*(1+i/100+M(FLB5,14)/100-M(FLB5,14)./100.*M(FLB5,9))));

FLB6=find(M(:,1)==1 & M(:,2)==0 & M(:,8)==1 & M(:,9)==0); % find large, savers with default and without agreement

M(FLB6,6)=0;

FLB7=find(M(:,1)==0 & M(:,2)==1 & M(:,8)==0); % find small, spenders without default

M(FLB7,6)=g*(30-(M(FLB7,4).*(1+i/100+M(FLB7,14)/100-M(FLB7,14)./100.*M(FLB7,9))));

FLB8=find(M(:,1)==0 & M(:,2)==1 & M(:,8)==1 & M(:,9)==1);  % find small spenders with default and agreement

M(FLB8,6)=g*(30-(M(FLB8,4).*(1+i/100+M(FLB8,14)/100-M(FLB8,14)./100.*M(FLB8,9))));

FLB9=find(M(:,1)==0 & M(:,2)==1 & M(:,8)==1 & M(:,9)==0); % find small spenders with default and without agreement

M(FLB9,6)=0;

FLB10=find(M(:,1)==0 & M(:,2)==0 & M(:,8)==0); % find small, savers, without default

M(FLB10,6)=s*(30-(M(FLB10,4).*(1+i/100+M(FLB10,14)/100-M(FLB10,14)./100.*M(FLB10,9))));

FLB11=find(M(:,1)==0 & M(:,2)==0 & M(:,8)==1 & M(:,9)==1);  % encontra países pequenos, poupados, com default e acordo

M(FLB11,6)=s*(30-(M(FLB11,4).*(1+i/100+M(FLB11,14)/100-M(FLB11,14)./100.*M(FLB11,9))));

FLB12=find(M(:,1)==0 & M(:,2)==0 & M(:,8)==1 & M(:,9)==0); % find small, savers, with default and without agreement

M(FLB12,6)=0;

FLB13=find(M(:,1)==1 & M(:,2)==2 & M(:,8)==0); % find large, ultrasavers, without default

M(FLB13,6)=u*(70-(M(FLB13,4).*(1+i/100+M(FLB13,14)/100-M(FLB13,14)./100.*M(FLB13,9))));

FLB14=find(M(:,1)==1 & M(:,2)==2 & M(:,8)==1 & M(:,9)==1);  % find large, ultrasavers, default and agreement

M(FLB14,6)=u*(70-(M(FLB14,4).*(1+i/100+M(FLB14,14)/100-M(FLB14,14)./100.*M(FLB14,9))));

FLB15=find(M(:,1)==1 & M(:,2)==2 & M(:,8)==1 & M(:,9)==0); % find large, ultrasavers, default, without agreement

M(FLB15,6)=0;

FLB16=find(M(:,1)==0 & M(:,2)==2 & M(:,8)==0); % find small, ultrasavers, default, without agreement
```

```
    M(FLB16,6)=u*(30-(M(FLB16,4).*(1+i/100+M(FLB16,14)/100-
M(FLB16,14)./100.*M(FLB16,9))));

    FLB17=find(M(:,1)==0 & M(:,2)==2 & M(:,8)==1 & M(:,9)==1);  % find
large, ultrasavers, default and agreement

    M(FLB17,6)=u*(30-(M(FLB17,4).*(1+i/100+M(FLB17,14)/100-
M(FLB17,14)./100.*M(FLB17,9))));

    FLB18=find(M(:,1)==0 & M(:,2)==2 & M(:,8)==1 & M(:,9)==0); % find
large, ultrasavers, default, without agreement


    M(FLB18,6)=0;

    M(:,5)=M(:,6);


 % SPREADS COMPUTATION

    M(:,12)=M(:,4)+M(:,6)-M(:,7)+M(:,10)-D(z)*c/(N-D(z));            %
disposable income

    M(:,13)=M(:,6)./M(:,12); % debt ratio

    FDH1=find(M(:,13)<=0.2);

    M(FDH1,14)=1*Q; % spread countries debt<20% %

    FDH2=find(M(:,13)>0.2& M(:,13)<=0.4);

    M(FDH2,14)=2*Q; %spread debt between 20% and 40%

    FDH3=find(M(:,13)>0.4& M(:,13)<=0.6);

    M(FDH3,14)=3*Q; %spread debt between 40% and 60%%


    FDH4=find(M(:,13)>0.6 & M(:,13)<=0.8);

    M(FDH4,14)=4*Q; %spread debt between 60% and 80%

    FDH5=find(M(:,13)>0.8);

    M(FDH5,14)=5*Q; %spread debt>80%


% RESULTS MATRIX

    M(:,12)=M(:,4)+M(:,6)-M(:,7)+M(:,10)-D(z)*c/(N-D(z));            %
disposable income

    YD(z)=sum (M(:,12)); % total disposable income

    YMD(z)=YD(z)/N; % average disposable income

    A(z)=sum (M(:,9)); % countries with agreement

    FYG=find(M(:,1)==1);

    YG(z)=sum(M(FYG,12)); % disposable income large countries

    YMG(z)=YG(z)/L; % average disposable income large countries

    FYP=find(M(:,1)==0);

    YP(z)=sum(M(FYP,12)); % disposable income small countries

    YMP(z)=YP(z)/(N-L); % average disposable income small countries
```

```
FYDF=find(M(:,8)==1);
YDF(z)=sum(M(FYDF,12)); % disposable income default countries
YMDF(z)=YDF(z)/D(z); % average disposable income default countries
FYDA=find(M(:,9)==1);
YDA(z)=sum(M(FYDA,12)); % disposable income agreement countries
YMDA(z)=YDF(z)/A(z); % average disposable income agreement countries
FYDX=find(M(:,2)==1);
X(z)=size(FYDX,1);
YDX(z)=sum(M(FYDX,12)); % spenders total disposable income
YMDX(z)=YDX(z)/X(z); % spenders average disposable income
FYDS=find(M(:,2)==0);
S(z)=size(FYDS,1);
YDS(z)=sum(M(FYDS,12)); % savers total disposable income
YMDS(z)=YDS(z)/S(z); % savers average disposable income

result(z,1)=YD(z)';
result(z,2)=YMD(z)';
result(z,3)=D(z)';
result(z,4)=A(z)';
result(z,5)=YG(z)';
result(z,6)=YMG(z)';
result(z,7)=YP(z)';
result(z,8)=YMP(z)';
result(z,9)=YDF(z)';
result(z,10)=YMDF(z)';
result(z,11)=YDA(z)';
result(z,12)=YMDA(z)';
result(z,13)=YMDX(z)';
result(z,14)=YMDS(z)';

%plot((1:z),result(:,1))% average disposable income
%title(['Average disposable income'])
%xlabel(['t '  ])
%plot((1:z),result(:,6),'g',(1:z),result(:,8),'r')    %    average
disposable income large vs small vs all     %title(['Average income:
large vs small countries'])
%legend('Large countries', 'Small countries');
%xlabel(['t '  ]);
```

```
    plot((1:z),result(:,10),'g',(1:z),result(:,12),'r')    %    average
income vs default vs agreement

    title(['Average income: default vs agreement countries'])

    legend('Default countries', 'Agreement countries');

    xlabel(['t '  ]);

    %plot((1:z),result(:,13),'g',(1:z),result(:,14),'r')   %    average
income vs spenders vs savers

    %title(['Average income: Spender vs savers'])

    %legend('Spenders', 'Savers');

    %xlabel(['t '  ]);

    %plot((1:z),result(:,3))

    %title(['Countries in default'])

    %xlabel(['t '  ]);


    end
```

4.9 Discussion

This paper was written shortly after European Central Bank started its quantitative easing program, buying bonds (sovereign and corporate) in the secondary market, in a period where the *troika* programs and the European countries bail-outs were still very present in the memory of European people. It is based in a Jean Tirole model, which is a game theory model to determine optimum strategies, and puts countries as the agents of the model. More precisely, the agents are governments that have to take decisions about borrowing. It is not a traditional macroeconomic model but it has a macroeconomic perspective. Not only because we are dealing with countries and governments, but also because we consider interdependences and spillovers typical of the global economy functioning.

One of the novel features of the model is the introduction of risk aversion at a macro level. In fact, this is typically an individual characteristic but we used here as a proxy of the stance of the governments regarding borrowing. We can interpret this risk aversion not as a particular feature of the Prime-minister or the minister of Finance himself but as a political preference of the country that can be conjuncture (after a crisis governments are more cautious, for example) or structural (like the Germany debt brake). This particular feature has already earned a citation[29] precisely because in the model “risk aversion is negatively correlated with both default probability and income”.

Among other conclusions, we find that higher risk aversion is associated with lower default levels, that default contagion can be is a very rapid process and monetary policy can prevent sovereign defaults through zero-interest rate policies but QE instruments have moral hazard problems.

The first version of the paper, presented in UECE 2017 conference and in ASEPELT 2017 Congress, did not have a central bank nor monetary policy analysis. Monetary policy was introduced in a later version, after some suggestions of the discussants in that conferences, with the objective of evaluating moral hazard phenomena and monetary policy impact (namely quantitative easing and ‘zero’ interest rates).

[29] Biondo, A. E. (2018). Learning to forecast, risk aversion, and microstructural aspects of financial stability. *Economics: The Open-Access, Open-Assessment E-Journal*, *12*(2018-20), 1-21

5. BRAIN DRAIN VERSUS BRAIN GAIN IN AN AGENT-BASED MODEL

Abstract

Brain drain phenomenon has been widely studied in Economics in the last five decades. The first papers tended to underline its negative effects and were focused mainly on underdeveloped or developing countries. Recent approaches, however, concluded that is possible to have a positive effect on the source economy related with human capital gains from skilled workers who become educated to emigrate but don´t have success in this objective. The overall effect on human capital accumulation and economic growth depends mainly on the education threshold needed to be accepted abroad and on the probability of emigration. In this paper, we present an agent –based to evaluate the beneficial brain drain scenario. With this agent-based model, which has some departures from the original model, we find that economic growth can benefit from a higher emigration probability and that risk aversion tends to mitigate the beneficial brain drain effect.

JEL: C63 E62 G01

5.1 Introduction

Brain drain is a phenomena usually seen as negative for the source economies – normally underdeveloped or developing countries – but this is, to say the least, a very narrow perspective. It is true that the brain drain can have harmful effects in the host country because it depletes human capital. But it can also represent an incentive to human capital acquisition. And, at least theoretically speaking, it can have a positive effect in economic growth through that mechanism. We can have two parallel dynamics which balance will determine the economic consequences of brain drain in the issuer economy. On one hand, the brain drain effect that measures the loss of human capital related to emigration of qualified workers. On the other hand, the brain gain effect that results from the fact that emigration works as an incentive to human capital accumulation but, because not all the qualified workers can emigrate, this represents a gain to the issuer economy.

There is a very wide range of literature about brain drain and, in particular, about the relation between brain drain and economic growth. Our aim, in this paper, is to evaluate this two opposite effects of brain in the context of an agent-based model. A model based on the one developed by Beine, Docquier and Rapoport (2001) but with additional heterogeneity of the agents and other stochastic components. The authors present a theoretical model to evaluate this potentially contradictory effects of brain drain and also an empirical analysis based on data for 37 developing economies that confirmed, in practice, "that the possibility of a beneficial brain drain could be more than a theoretical curiosity".

Our paper is organized as follows: section 2 provides a brief literature review about brain drain, its mechanisms and economic consequences and previous agent-based models or simulations to deal with it; section 3 details the original Beine, Docquier and Rapoport (2001) model; section 4 presents our agent-based model and the simulation details, assumptions and scenarios; section 5 presents the simulation results and section 6 concludes.

5.2 Literature review

Brain drain is a topic with a long tradition in economics[30]. We can find examples in the economic literature at least in the last 50 years. Three waves of research can be identified. The first wave, dating back to the 60´s, was focused mainly on welfare analysis and was based on the international trade theoretical framework. Because of this trade oriented approach, in a context of some free trade apology, these 'early' papers tend to conclude in favor brain drain contribute to world economy with relatively low or even no negative effect in the source economy.

Grubel and Scott (1966) is one of the most cited papers about brain drain from those days. It was justified, as their authors put it, "by recent strong manifestations of public interest in two major problems in international relations: first, the migration of highly skilled individuals to the U.S.-often referred to as the "brain drain"-and, second, the large- scale program of training foreign students in the U.S". And it concludes that, among other things, that "emigration of highly skilled persons reduces the welfare of the remaining people only under rather rare circumstances, we can make a good case for the proposition that these types of emigrants in fact tend to increase the welfare of their former countrymen in several important ways". Such as remittances but also through influencing domestic polices and institutions.

Johnson (1967), for instance, adopts a liberal perspective in opposition to a nationalistic view in favor of high skilled emigration.

Berry and Soligo (1969) centered their analysis on the welfare implications of international migrations to detail the conditions under which loss to the remaining population will occur. They conclude that, in general, brain drain has negative consequences to source economy but that is possible to find "few cases where gain (or no change) may result".

In the 1970´s, a new wave – the second - pioneered by Jahdish Bhagwati and others (Bhagwati and Hamada, 1974; Bhagwati and Rodriguez, 1975; Bhagwati, 1976) focused on the welfare consequences of the brain drain. In

[30] For an extensive review of brain drain research see, for example, Docquier and Rapoport (2011).

this period, many papers were published regarding different institutional settings of brain drain.

The third stream it's the most directly related to this paper. It is concerned with the balance between positive and negative effects of brain drain and the determination of the conditions for each scenario. It began in the 1990 and it has theoretical but also empirically based, tough data is not always enough in terms of quality and quantity. One of the main examples of these approach is precisely Beine, Docquier and Rapoport (2001) paper that we use in our agent-based model formulation.

But there are of course many other interesting examples. The same Beine, Docquier and Rapoport (2008) tested again the beneficial brain drain hypothesis based on a theoretical model – different from 2001 version but similar in conceptual terms – but also on a new data set of emigration rates by educational level[31]. In an analysis for 127 countries, the three authors found a positive effect in human capital formation based on the idea that, because not all qualified workers that intend to migrate are able to do it, there is a home gain in terms of human capital. More precisely, they estimate that "the elasticity of human capital formation to skilled migration is equal do about 5% and is very stable across specifications and estimation methods". Countries with "low levels of human capital and low skilled emigration rates are more likely to experience a beneficial brain drain".

Agrawal, Kapur, McHale and Oettl (2011) concluded that emigration of skilled workers may weaken local knowledge networks but innovation capacity may be benefited from e linkages to human capital accumulated abroad. The paper presents a theoretical approach to determine the optimal equilibrium between local knowledge and diaspora human capital but also an empirical application to India´s economy.

Dustmann, Fadlon and Weiss (2011) presents a model where workers can have two different skills that can be augmented in a learning by doing process at home or abroad and where workers have the possibility of returning home after migration. In this case, brain drain effect appears as a result of the return of skilled workers that had studied abroad and take back

[31] The data set was first presented by Docquier and Marfouk (2006) and used also in Docquier, Lohest and Marfouk (2006).

their knowledge to home. The paper discusses the incentives to return based on investment considerations, namely the return on human capital.

On this topic, Biondo, Pluchino and Rapisarda (2012) has one of the attempts to model brain drain through an agent-based model focusing precisely the return scenario. Based on computational simulations regarding two individual features: risk aversion and initial expectation. Basically, they analyze “the comparison between prospective life in home country and prospective life abroad, period by period, so that the final choice can be simulated as a result of the comparison between levels of expected utility in both locations”. Results points out “that the final decision of an agent strictly depends on the ratio between these two individual features: if the risk aversion is very high with respect to the initial expectation, the return decision occurs with high probability and vice-versa”.

Klabunde and Willekens (2016) presents an extensive review of ABM applied to migrations, namely the decision making rules. It provides also very useful criteria that should be used in ABM design for migration analysis and poses two reflection questions: which decision theory to choose and what is the role of the data?

There are several channels through which the brain drain phenomena has consequences to the source and host economies. Channels such as the human capital, remittances or international networks effects. In this paper we are interested, in particular, in human capital and its implications in economic growth of the source economy.

5.3 The model

Before presenting in detail our ABM it could be very useful to understand the dynamics of the Beine, Docquier and Rapoport (2001) model in which our ABM is based. This is an overlapping generation model where the agents live two periods. There are young and old agents. In the first period, agents decide how much time they spend studying based on their emigration prospects. There is a minimum educational level threshold to emigrate. In the second period, the unqualified agents stay in the country working with a productivity determined by the human capital investment made in the previous period and with it specific ability to learn. Part of the qualified workers, i.e. those with a minimum educational level, will emigrate (the brain drain effect) but some will remain in the country working (the brain gain effect).

Heterogeneity of the model lies in the fact that each agent has a specific ability to learn that determines his future productivity. Economic growth is based on the intergenerational transmission of human capital: every young worker 'come to the world' with the average human capital level of the older workers of the previous period.

5.3.1 Production

Production in period t is given by production function with constant returns to scale where the representative firm uses capital K_t and labor H_t to produce a composite good:

$$Y_t = F(K_t, H_t) \tag{1}$$

$$y_t = f(k_t) \tag{2}$$

where Y_t is the production in period t and y_t is a fuction of capital per efficient hour worked ($y_t = \frac{Y_t}{H_t}$) with $k_t = \frac{K_t}{H_t}$.

The representative firm operates in a competitive environment where the input prices are determined by their marginal productivity. Interest rate r is defined fixed in the international market and it is given, what means that this economy is small enough to affect the international financial markets. Stock of capital per efficient hour worked and wages are determined based on this interest rate. For the sake of simplicity, we are assuming a constant interest rate and wage normalized to 1.

5.3.2 Agents decision

Each young agent i has a human capital endowment h_t in period t that came from previous period. In his first period of life, the agent decides how much time he will allocate to education. It is a discrete option between $e_t^i =$

$\bar{e}$, with $\bar{e}$ being considered the minimum threshold to emigrate, or nothing ($e_t^i = 0$). Each agent has a specific ability to learn, randomly distributed, which is crucial to human capital accumulation.

Human capital level of agent i in the second period (h_{t+1}^i) is a function of time spent studying and individual ability to learn. In particular:

$$h_{t+1}^i = [1 + a^i e_t^{i\beta}]h_t \quad (3)$$

with $0 < \beta < 1$ and a^i is the individual ability parameter uniformly distributed in the interval o $[\underline{a}, \overline{a}]$.

The agent decision depends on the comparison between domestic and international return on human capital. Because we are analyzing the brain drain phenomena, we assume that international return is bigger.

We are considering w (with $w > 1$), the relative return of education net of emigration costs (monetary and other costs), as given. Which means that there is no room for productivity convergence and that domestic/external technological are a constant and migrations are not big enough to have consequences in the host country productivity.

The migrant agent productivity in the second period is given by:

$$h_{t+1}^i = [1 + wa^i e_t^{i\beta}]h_t \quad (4)$$

Achieving a minimum education level of $e_t^i = \bar{e}$ is a necessary but not sufficient condition to emigrate. Only part of the potential emigrants – i.e. those with an education level above de threshold – migrates. There is uncertainty in this process namely, the agents face a p probability of emigration and a $(1 - p)$ probability of stay 'in home'. This probability is associated a factors that normally affects the migratory flows, such as regulations, restrictions in the host countries and other political decisions that affects migrations.

We are admitting also that this probability does not depend on the individual ability to learn of the agents[32].

Agents are risk neutral and maximize their lifetime expected return:

$$E\left[h_t\left(1-e_t^i\right)+\left(\frac{h_{t+1}^i}{1}+r\right)\right] \tag{5}$$

With r as the intertemporal discount rate. Agent i invests in education if:

$$h_t(1-\overline{e})+\frac{p[1+wa^i\overline{e}^\beta]h_t}{1+r}+\frac{(1-p)[1+a^i\overline{e}^\beta]h_t}{1+r}\geq h_t+\frac{h_t}{1+r} \tag{6}$$

The part of the population that decides to become educated is determined from the indifferent agent:

$$a_i \geq a_E \equiv \frac{\overline{e}^{1-\beta}(1+r)}{\Phi(p,w)} \tag{7}$$

where $\Phi(p,w)=1+p(w-1)$ is defined in the interval [1 ,w] and a_E is the critical agent´s ability.

When emigration is not possible ($p=0$), critical agent´s ability is given by:

$$a_F \equiv \overline{e}^{1-\beta}(1+r) \tag{8}$$

And the proportion of educated agents i the population is:

[32] Considering two different education levels related with different probability of emigration can be an important and interesting innovation in the model for ABM simulation purposes.

$$P_F = max\{0, (\overline{a} - a_F)/(\overline{a} - \underline{a})\} \quad (9)$$

On the opposite, when emigration is a certain outcome ($p = 1$), there will be more people investing in education. Critical ability is $a_M \equiv a_F/w$ and the proportion of educated agents in the population is zero ($P_M = 0$).

In the intermediate situation, when the emigration probability is between 0 and 1, critical agent is determined by $a_M < a_E < a_F$ and the proportion of educated agents in the remaining population is:

$$P_E = max\{0; \frac{(1-p)(\overline{a}-a_E)}{a_E-\underline{a}+(1-p)(\overline{a}-a_E)}\} \quad (10)$$

With P_E being higher or lower than P_F.

5.3.3 Migration effects

To evaluate the impact of migrations in economic growth, we consider *per capita* income. This option allows us to overcome population variations interference. Because emigrants are randomly chosen among educated agents, educated composition among the emigrants and the agents that sty home is the same. There, average human capital level of the remaining population is determined by:

$$h_{t+1} = \frac{\overline{a}-\underline{a}}{a_E-\underline{a}+(1-p)(\overline{a}-a_E)} \left[\int_{\underline{a}}^{a_E} h_t U(a)da + (1-p)\int_{a_E}^{\overline{a}} \left(1 + a\overline{e}^{\beta}\right) h_t U(a)da\right] \quad (11)$$

with $U(a)$ corresponding to the uniform distribution $[\underline{a}, \overline{a}]$.

From the previous expression, we can determine the equilibrium growth path of the economy:

$$g_{t+1} = \frac{h_{t+1}-h_t}{h_t} = \frac{(1-p)\overline{e}^{\beta}(\overline{a}^2-a_E^2)}{2[a_E-\underline{a}+(1-p)(\overline{a}-a_E)]} \tag{12}$$

Is this expression we can clearly identity the two opposite effects of brain drain in economic growth. *Drain effect* is related to p and , the higher this parameter is, the higher is the negative effect ($\frac{\partial g}{\partial p} < o$ for a_E constant). On the other hand, equilibrium growth rate is a decreasing a_E function that is, also, a p decreasing function. This is the *brain effect*. Global effect depends on the balance of these two components.

To determine the necessary conditions for a positive brain drain we will start with a closed economy ($p = 0$) and assume also that $\underline{a} = 0$. We will have:

$$g_F = \frac{\overline{e}^{\beta}(\overline{a}^2-a_F^2)}{2\overline{a}} \tag{13}$$

$$a_F = \overline{e}^{1-\beta}(1+r) \tag{14}$$

where g_F represents economic growth rate in an economy without migrations e a_F the critical agent ability in that economy.

Because our goal is the evaluation of the global impact of migrations on economic growth. More precisely, we want to compare growth rates with and without migration option. In formal terms, we have a beneficial brain drain (BBD):

$$\frac{(1-p)\overline{e}^{\beta}(\overline{a}^2-a_E^2)}{2a_E+2(1-p)(\overline{a}-a_E)]} > \frac{\overline{e}^{\beta}(\overline{a}^2-a_F^2)}{2\overline{a}} \tag{15}$$

where $a_E = a_F/\,\Phi(p,w)$.

Given the fact that we don´t have economic growth in autarky ($a_F = \overline{a}$), opening borders is never a bad option in terms of economic performance. This is why we exclude this extreme case and impose an interior solution ($a_F < \overline{a}$).

We have a beneficial *brain* drain if, and only if, migration probability verify the following condition:

$$p \times Z(p) = p(Ap^2 + Bp + C) < 0 \qquad (16)$$

$$A = (w-1)^2 \qquad (17)$$

$$B = (w-1)\left(\frac{\overline{a}^2 - a_F^2}{\overline{a}a_F} + 3 - w\right) \qquad (18)$$

$$C = \frac{\overline{a}^2 - a_F^2}{\overline{a}a_F} - 2(w-1) \qquad (19)$$

For even p above 0, this condition depends on the sign of $Z(p)$. $Z(\varepsilon) \cong C$ for a low and positive or negative ε. On the opposite, $Z(1) = w(\overline{a}^2 - a_F^2)/(\overline{a}a_F)$ is always non-negative, which means that when migration probability is 1 it is always prejudicial for the country.

Between these two extreme points, the overall effect depends on the signs of B and C. C is positive if human capital investment is relatively high in the closed economy (i.e., if a_F is low enough). In this case, *brain drain* has always negative impact if $B > 0$ or positive impact in a small intervall for for the migration probability if $B < 0$. When $B > 0$, intuition is as follows: emigrants are choosen among educated agents that would be educated even in a closed economy. When $B < 0$, migration probability must be high enough to produce a *brain effect* but low enough to avoid a significant *drain effect*.

5.4 ABM and simulation

5.4.1 Agent-based model

Our agent-based model is based on Beine, Docquier and Rapoport (2001) original model with three main departures. First, our economy has several periods instead of only two tough each agent lives only for two periods: the first period as a young agent when he will have to decide to study or not based on his expectations of future income and the cost associated; and the second period as an old agent that can stay at home or emigrate depending on his qualification and the emigration opportunities. In each period, the young agents are the new entrants in the active population that is growing and also the old agents that are replaced by young agents. We are working in an overlapping generation environment.

Second, active population grows at a g rate. This means that, in each period, population variation is negatively affected by emigration but positively affected by new entrants in the labor market. These new entrants are young agents which have the same human capital endowment of the old agents that became young.

Active population evolution in period $t+1$ is given by:

$$N_{t+1} = N_t(1+g) - M_{t+1} \tag{20}$$

where M_{t+1} is the emigration in period t+1 and g is the active population growth rate. For the sake of simplicity, we assume that population is constant which means that, in each period, population growth is exactly the number of emigrants.

Three, agents have two different risk aversion levels: a normal one which simply demands that expected return from migration surpasses return without migration and a higher risk aversion agents that demand a v premium for migration. In mathematical terms, the normal risk aversion agent decides to study if:

$$h_t(1-\overline{e}) + \frac{p[1+wa^i\overline{e}^\beta]h_t}{1+r} + \frac{(1-p)[1+a^i\overline{e}^\beta]h_t}{1+r} \geq h_t + \frac{h_t}{1+r} \qquad (21)$$

But higher risk aversion agents demands a higher return:

$$h_t(1-\overline{e}) + \frac{p[1+wa^i\overline{e}^\beta]h_t}{1+r} + \frac{(1-p)[1+a^i\overline{e}^\beta]h_t}{1+r} \geq (1+v)(h_t + \frac{h_t}{1+r}) \qquad (22)$$

Risk aversion is one of the two sources of individual heterogeneity in this model and it is randomly attributed. The other one is agent´s individual learning capacity. These two variables together produces an individual human capital level for each agent and also different inputs for education decision.

For the purpose of economic analysis we are more interested in individual production – or human capital – instead of total income because it is a way of overcoming the effect of a changing population. This doesn´t mean, however, that we disregard completely the aggregate effect. We use average human capital evolution to assess the human capital consequences of brain drain in terms of average level in our economy. And we consider too the overall production to determine the aggregate consequences of the brain drain phenomena and its dynamics in more extreme scenarios.

In practical terms, we analyze brain drain effect comparing the different scenarios for emigration probability (p) and risk aversion premium (v). Our aim is to determine the relation of the different variable levels with brain drain and brain gain effects. Remember that, as was said earlier, the overall effect depends heavily on the trade-off between individual ability and emigration probability. In our ABM, we consider other variables but these two play an important role in our scenarios.

Average human capital in period t in the economy is given by:

$$h_t = \frac{\sum_{i=1}^{N} h_{i,t}}{N} \qquad (23)$$

Overall production is given by:

$$Y_t = F(K_t, H_t) \tag{24}$$

Because we are mainly interested in human capital we assume that capital stock K is constant – i.e. that investment are only strictly enough to compensate depreciations – and the technology is also the same along the time path. Production function Y_t follows a Cobb-Douglas form with a stochastic error term e such as:

$$Y_t = AH_t^{\alpha}K_t^{\beta} + e \tag{25}$$

with $0 < \alpha, \beta < 1$ and e as a random shock. The constant population hypothesis means that, instead of using aggregate human capital H_t, we use average human capital times 1000 agents.

5.4.2 Simulation dynamics

For ABM simulation we consider P periods. There are N agents in the economy belonging to four different groups: young agents studying; young agents not studying; old skilled agents and old unskilled agents. Some of the old skilled agents will became emigrants. In period t, young agents decide about studying based on their individual skills and on the expected migration return. In period $t + 1$, educated agents might become emigrants depending on a random selection among this educated group with an education threshold above the minimum threshold required abroad – this might correspond to a BsC degree, a MsC degree or even a PhD. We don´t differentiate qualifications, only levels.

Older agents without education and older educated agents that are not ‘selected’ to emigration remain in the country. They have different productivity (or human capital) based on their different skills and education. Young agents will have a human capital that comes from old agents of the previous period, but some decide to study which represents a cost (e) and the others don’t.

The young agent i human capital in period t is given by:

$$h_t^i = h_{t-1}^j - e \qquad (26)$$

where h_{t-1}represents previous period human capital and e is the education cost for studying agents (the other have $e = 0$).

The agent i human capital in period $t + 1$ is given by:

$$h_{t+1}^i = [1 + wa^i e_t^{i\beta}]h_t^i \qquad (27)$$

Non-migrants have $w = 1$ and agents that decided not to study have $e = 0$. This means that migrants have the highest human capital, followed by agents that have studied in the previous period but stayed at home. Agents that did not study have same human capital of the previous period.

5.4.3 Scenarios and parameters

For our simulations we considered six different scenarios. A baseline, four variations for parameter p, which plays a crucial role in brain drain consequences, and a sixth scenario with a higher risk aversion coefficient (v), as summarized in table 5.1. We consider a constant population. In our baseline, we have 100 periods, long enough to provide steady state results, 1000 initial agents (300 young agents and 200 high risk aversion agents), a 10% emigration probability ($p = 0.1$), 2 for parameter b, a 2% discount rate ($r = 0{,}02$), 0.3 for risk aversion parameter (v), and, respectively, 10, 50, 0.5 and 0.5 for Cobb-Douglas production function parameters.

The other scenarios have some changes from the baseline:

- scenarios 2 to 5 have increasing emigration probabilities (from $p = 0{,}3$ until $p = 0.9$)
- scenario 6 considers an higher risk aversion parameter ($v = 1$)

<table>
<tr><th></th><th>P</th><th>N</th><th>J</th><th>H</th><th>p</th><th>b</th><th>r</th><th>w</th><th>e</th><th>v</th><th>A</th><th>K</th><th>alfa</th><th>beta</th></tr>
<tr><td>Baseline</td><td rowspan="6">100</td><td rowspan="6">1000</td><td rowspan="6">300</td><td rowspan="6">200</td><td>0.1</td><td rowspan="6">2</td><td rowspan="6">0.02</td><td>1.5</td><td>0.2</td><td>0.3</td><td rowspan="6">10</td><td rowspan="6">50</td><td rowspan="6">0.5</td><td rowspan="6">0.5</td></tr>
<tr><td>Scenario 2</td><td>0.3</td><td>1.5</td><td>0.2</td><td>0.3</td></tr>
<tr><td>Scenario 3</td><td>0.5</td><td>1.5</td><td>0.2</td><td>0.3</td></tr>
<tr><td>Scenario 4</td><td>0.7</td><td>1.5</td><td>0.2</td><td>0.3</td></tr>
<tr><td>Scenario 5</td><td>0.9</td><td>1.5</td><td>0.2</td><td>0.3</td></tr>
<tr><td>Scenario 6</td><td>0.1</td><td>1.5</td><td>0.2</td><td>1</td></tr>
</table>

Table 5.1 – Simulation scenarios

5.5 Results

In the baseline scenario, GDP follows an increasing trend with some fluctuations related with its function design and, in particular, its random term. Figure 5.1 depicts a typical run for the Baseline Scenario.

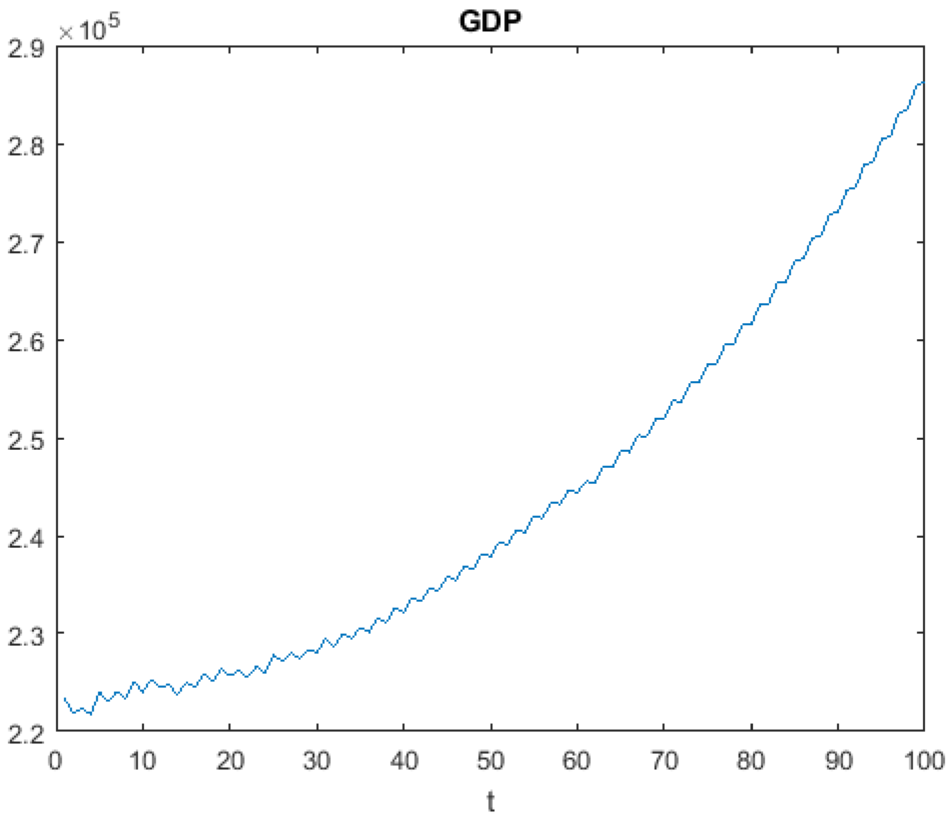

Figure 5.1 – Baseline Scenario: GDP

Average human capital followed a different pattern. First, in an concavous configuration and after, around $t = 50$, turning to a convex behavior. Figure 5.2 presents the result of a typical run:

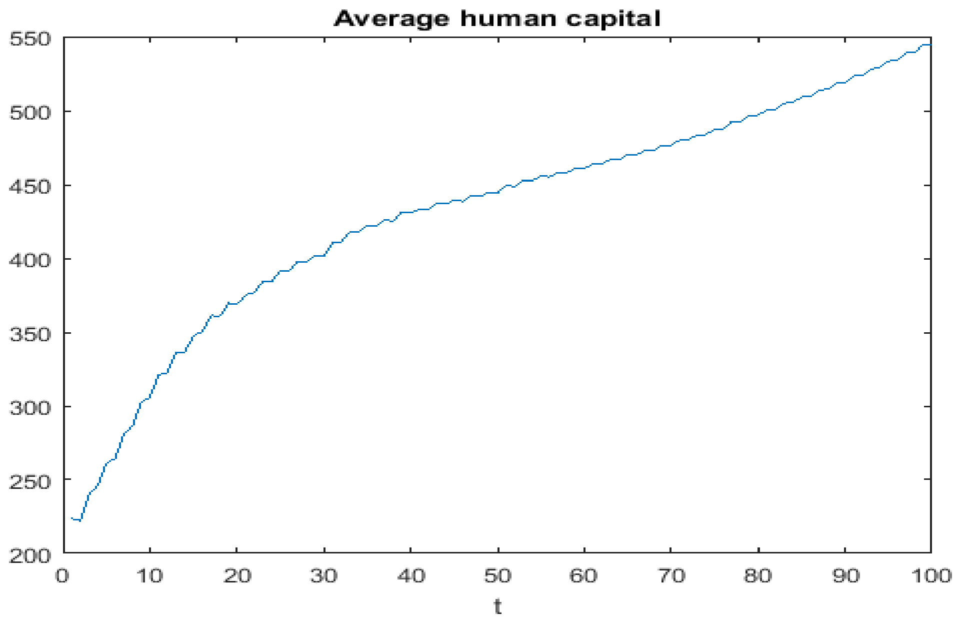

Figure 5.2 – Baseline Scenario: Average human capital

We can see also the simulation results for the number of emigrants in the baseline scenario in Figure 5.3:

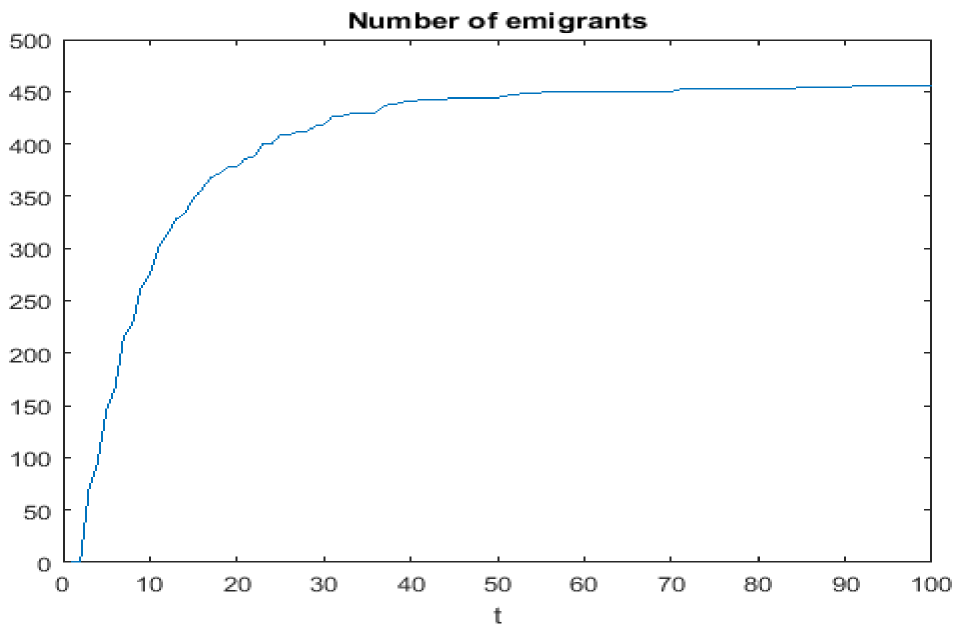

Figure 5.3 – Baseline Scenario: number of emigrants

To compare economic performance for the different scenarios, we run them 30 times and computed the averages. Table 2 presents the main results steady state for GDP, average human capital and total emigrants:

	Y growth (%)	*Human capital growth (%)*	*Final emigrants*
Baseline	24	54	465
2	20	44	427
3	19	42	452
4	19	42	487
5	0	0	0
6	19	43	395

Table 5.2 – Scenario comparison after 30 runs

We can draw some conclusions from this results. First, p plays a crucial role in determining brain drain economic effects. Human capital growth tends do decrease with increases in p, with an extreme scenario above 0.9 (or roughly that level) where there is no human capital gain at all. This means that all the educated agents have the opportunity to emigrate and, for that reason, the source economy does not have any benefit. This is a conclusion that probably applies in countries with very low percentage of qualified workers and where many or all of them can emigrate if they decide to do it.

Second, the final number of emigrants don´t have a linear relation with p, a behavior that is probably related with the intrinsic characteristics of the emigration decision process and the random factors of the model. There are 465 emigrants with $p = 0.1$, 427 when the probability is 0.2 and 487 for $p = 0.7$.

We can easily see the differences of human capital accumulation in a typical run as depicted in figure 5.4 with the results for the six scenarios:

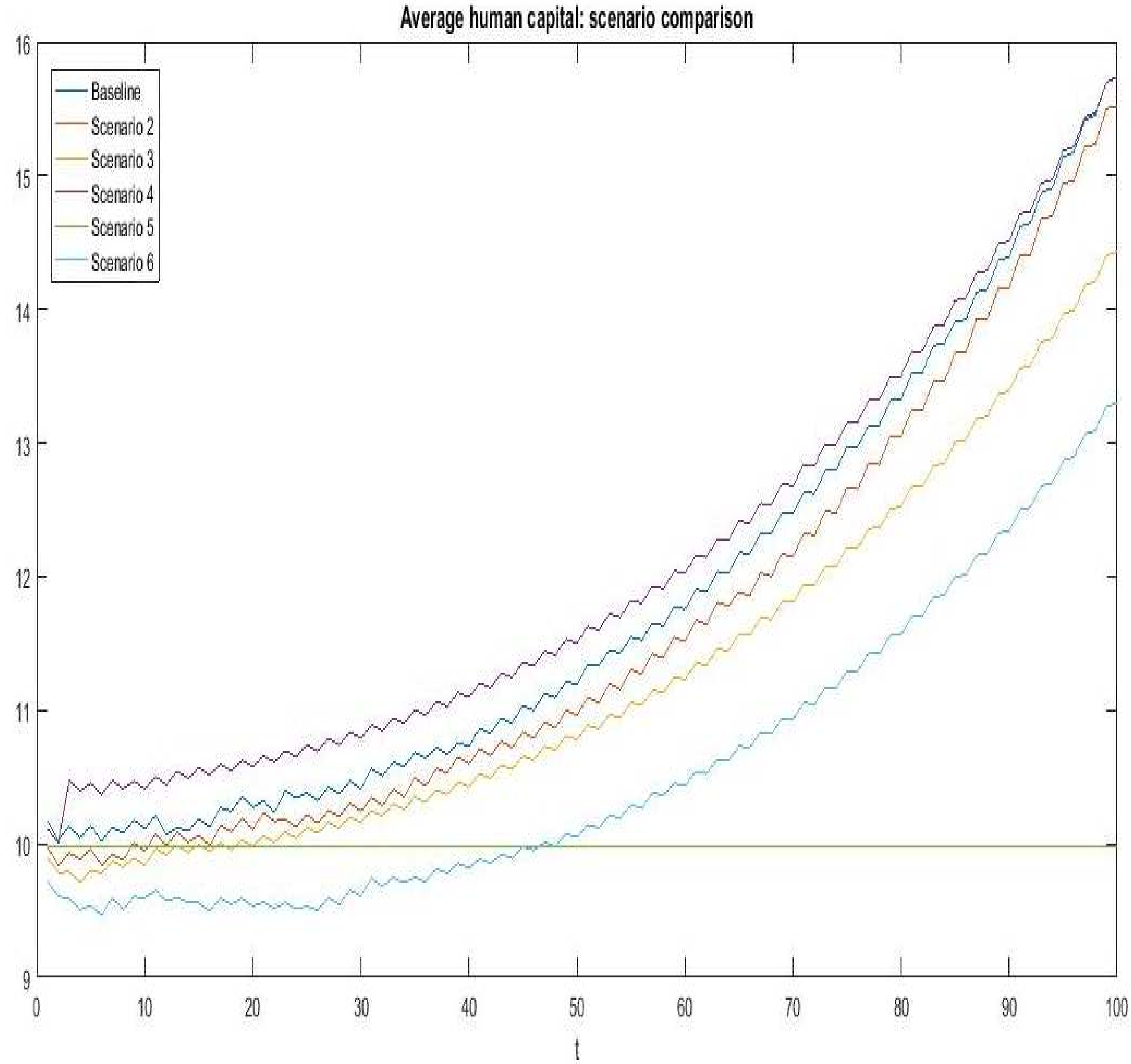

Figure 5.4: Average human capital in different scenarios

This results show that changes in emigration probability p can have meaningful effects on human capital accumulation and, hence, in economic growth too. As we said earlier, this parameter has opposite consequences in terms of economic performance. On one hand, higher p values are an incentive for human capital accumulation because individual have a higher emigration expected income but, on the other hand, this higher p represents a larger fraction of skilled workers that leave the country each year. When we compare Baseline Scenario with Scenarios 2, 3 and 4 it is precisely this kind of effect that is working.

Scenario 3 have a higher risk aversion parameter for higher risk aversion agents and, for that reason, it has a lower total economic growth after 30 periods and also a lower number of emigrants. This is the consequence of the fact that risk aversion agents demands a higher return of emigration to study and it has consequences in human capital accumulation.

If we don’t assume the constant population hypothesis and don´t have any population growth ($g = 0$), the economy will be severely affected by brain drain. In the first 10 to 20 periods, GDP decreases and, after it, human capital accumulation starts to become effective and the income growth will turn positive. But that is not enough to compensate the first negative years and GDP will finish lower than in the first period, for all the scenarios considered, as represented in figure 5.5:

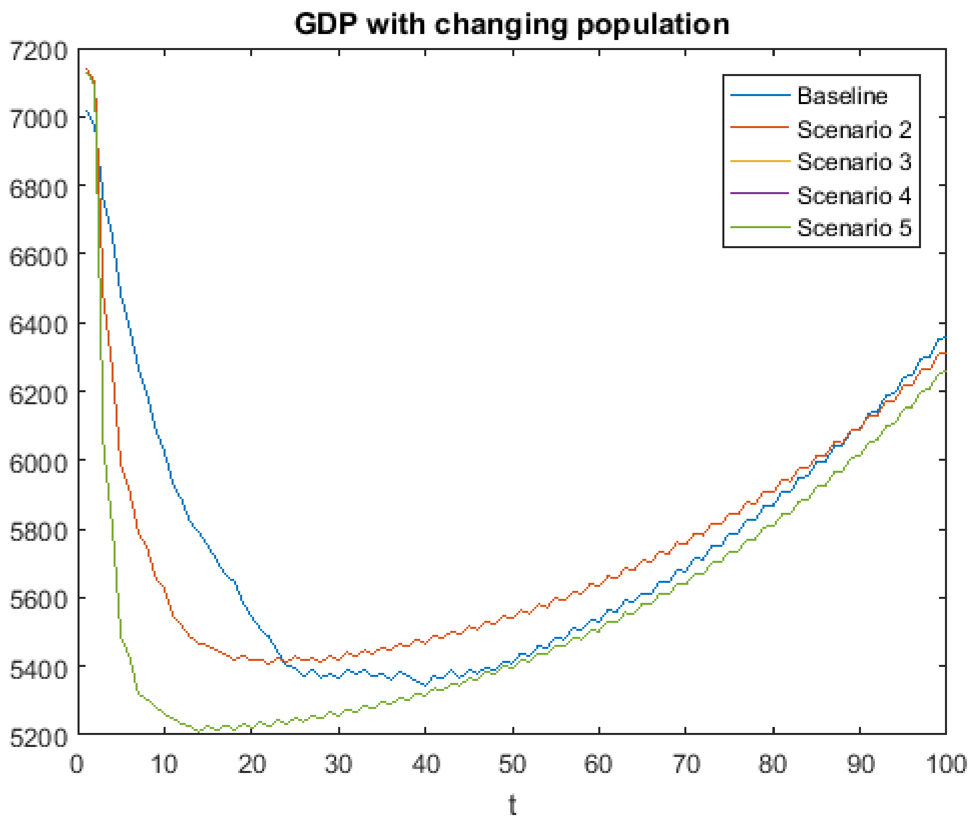

Figure 5.5: GDP with zero population growth

5.6 Concluding remarks

In this paper we presented an agent-base model based on Beine, Docquier and Rapoport (2001) to determine brain drain effects in economic growth. In particular, our aim was to evaluate the beneficial brain drain hypothesis, i.e., the possibility of brain drain having positive effects on the source economy through human capital accumulation.

We simulate six scenarios for different emigration probability and risk aversion levels and we conclude that:

1) as in the original theoretical model, emigration probability is a crucial parameter to determine brain drain consequences
2) economic growth can be higher or lower depending on the specific emigration probability p with a non-linear relation between them
3) higher individual risk aversion is related with lower economic growth and a lower beneficial brain drain effect

This paper is just a first attempt to construct an agent-based model for brain drain analysis. There are many ways of improving it: in terms of individual heterogeneity (different qualifications, for example, with different emigration probability); business cycles (generating, for instance, cycles and endogenous unemployment that can be used to change individual propensity to emigrate); or world economy (having, for example, some dynamics in the world business cycle and not just a static situation).

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5.8 ODD protocol

5.8.1 Overview

Purpose: The model aims to analyze beneficial brain drain hypothesis in an overlapping generation economy with heterogeneous agents facing uncertainty about emigration prospect. It is adapted from Beine, Docquier and Rapoport (2001) with additional heterogeneity (risk aversion levels) and stochasticity (agent´s individual characteristics).

State variables and scales:

The agents have three different individual characteristics: age (junior or senior), risk-aversion levels (high-risk or low-risk) and ability to learn (randomly generated and source of heterogeneity).

The agent´s parameter are:

- β (education effect in human capital accumulation)
- e (education cost)
- v (risk aversion)

The state variables are:

- p (emigration probability)
- r (discount rate)
- w (emigration premium)
- A, K, α, b (Cobb-Douglas parameters)

Process overview and scheduling: In each period, the junior agents decide about studying based on their expected return in the future. Those who opt to study, have a cost associated to that option. The senior skilled agents can have the possibility of emigration but the selection is random and, in the end, some of them have to stay. Human capital is updated and population grows older (young to senior and senior to young).

5.8.2 Design concepts

Emergence: Individual decisions and emigration probability that each agent faces provides a bottom-up effect where local and individual inputs have aggregate consequences. Namely, individual agent decisions regarding education will result in the emergence of macro patterns in terms of human capital accumulation and GDP growth.

Adaptation: Some of the agent´s characteristics change in every period, in particular, their age (young or old), studying approach or emigration status. The parameters are always the same but the changing environment provides a different context to which the agents have to adapt

Fitness: Performance evaluation is made at an aggregate level tough it is based, in first place, on average human capital as a proxy of human capital gains. We also compute GDP to have a macro perspective of the emigration impact on economic long-term performance.

Prediction: Education decision depends on the expected human capital accumulation because emigration is not granted even for skilled workers.

Interaction: Agents interact directly between them because new junior agents are the previous period senior agents and there is a human capital legacy being passed through. Indirectly, each individual decision has consequences in GDP growth and human capital supply of the economy.

Stochasticity: The main source of stochasticity in the model is the emigration probability p. This parameter is crucial to determine the economic impact of brain drain in the source economy. Other sources of stochasticity are the initialization of model, where all the agents are randomly generated and individual ability is itself a random variable, and the random shock of the Cobb-Douglas production function.

Collectives: Student versus non-students are the most relevant groups under analysis, tough there is at least other one that can be considered: junior versus senior.

Observation: Economic performance, GDP growth and human capital accumulation (total and average and group) are the outputs of the model. This metrics are presented numerically but also graphically.

5.8.3 Details

Initialization: The characteristics of the agents are set randomly, including initial human capital with values ranging from 0 to 20.

Input: Simulations considered 1000 agents (300 young and 200 high-risk aversion), 40 periods and we assume a constant population. In the baseline scenario, we set a 10% emigration probability ($p = 0.1$), 2 for parameter b (that affects education contribution to human capital formation), a 2% discount rate ($r = 0{,}02$), 0.3 for risk aversion parameter (v), and, respectively, 10, 50, 0.5 and 0.5 for Cobb-Douglas production function parameters.

5.9 Appendix: Matlab

```
function [resultado]=drainbase(P,N,J,H,p,b,r,w,e,v,A,K,alfa,beta)
% P= periods
% N=initial number of agents
% J=initial number of junior agents
% H=number of high risk aversion agents
% p=emigration probability
% b=human capital parameter beta
% r=discount rate
% w=emigration premium
% e=education emigration threshold
% v=high risk aversion parameter
% g=population growth
% A=cobb douglas parameter
% K=cobb douglas capital
% alfa=labor parameter cobb douglas
% beta=capital parameter cobb douglas
```

```
% run for baseline
resultado=drainbase(100,1000,300,200,0.1,2,0.02,1.5,0.2,0.3,10,50,0.5,
0.5);


% Initiation of agents characteristics

M=zeros(N,11);  % 1 Junior, 2 high risk aversion, 3 ability, 4
students, 5 emigrants, 6 previous period human capital, 7 current
human capital, 8 total human capital for agents that stay, 9 future
human capital no emigration, 10 future human capital with emigration,
11 expected human capital


Mrand=randperm(N);

M(Mrand(1:J),1)=ones(J,1); % junior agents

Mrand=randperm(N);

M(Mrand(1:H),2)=ones(H,1); % high risk aversion agents

Mrand=randperm(N);

M(Mrand(1:N),3)=rand(N,1); % ability

M(:,6)=20*rand(N,1); % initial human capital = 20

M(:,8)=M(:,6)+M(:,6)./(1+r); % human capital in t and t+1

M(:,9)=M(:,6).*(1+M(:,3).*e.^b); % next period human capital no
emigration

M(:,10)=M(:,6).*(1+w*M(:,3).*e.^b); % next period human capital with
emigration

M(:,11)=(p.*M(:,10))/(1+r)+((1-p).*M(:,9))/(1+r); % expected human
capital for educated agents


% cicle


for z=1:P


% EDUCATION DECISION


FE=find(M(:,1)==1 & M(:,11)>=(1+v*M(:,2)).*M(:,8)+e*M(:,6)-M(:,6)); %
find juniors that decide to study

M(FE,4)=1; % define junior students


% HUMAN CAPITAL UPDATE


FJE=find(M(:,1)==1 & M(:,4)==1 & M(:,5)==0); % find junior students
and non-emigrantes (senior the replace)

M(FJE,7)=M(FJE,6)-e; % previous period human capital - education cost
```

```
FJNE=find(M(:,1)==1 & M(:,4)==0 & M(:,5)==0); % find junior no
students and no emigrants (senior they replace)

M(FJNE,7)=M(FJNE,6); % previous period human capital

FVE=find(M(:,1)==0 & M(:,4)==1 & M(:,5)==0); % find senior, students
and no emigrants

M(FVE,7)=M(FVE,6).*(1+M(FVE,3).*e.^b); % previous period human capital
+ education gain previous period human capital

FVNE=find(M(:,1)==0 & M(:,4)==0 & M(:,5)==0); % find senior, no
students and no emigrants

M(FVNE,7)=M(FVNE,6); % previous period human capital or

M(:,6)=M(:,7);% human capital updtate


M(:,8)=M(:,6)+M(:,6)./(1+r); % human capital in t and t+1

M(:,9)=M(:,6).*(1+M(:,3).*e.^b); % next period human capital no
emigration

M(:,10)=M(:,6).*(1+w*M(:,3).*e.^b); % next period human capital with
emigration

M(:,11)=(p.*M(:,9))/(1+r)+((1-p).*M(:,8))/(1+r); % expected human
capital


% EMIGRATION


FNX=find(M(:,1)==0 & M(:,4)==1 & M(:,5)==0); % find senior, skilled
and non-emigrants

EP=size(FNX,1); % number of potential emigrants

X=ceil(EP*p);% compute emigrants number in the period

FNXrand=randperm(EP); % random senior

M(FNXrand(1:X),5)=1; % define emigrantes

E(z)=sum(M(:,5));


% AGEING POPULATION


M(:,1)=M(:,1)*-1+1; % junior become senior and senior become junior

FV=find(M(:,1)==1);% find junior agents

M(FV,4)=0; % clean junior agents to decide about studying


% RESULTS MATRIX


S(z)=sum (M(:,4)); % number of students

FXP=find((M(:,5)==0)); % find non emigrants

H(z)=sum(M(FXP,7)); % total human capital
```

```
HM(z)=H(z)/size(FXP,1); % average human capital humano

PIB(z)=A*1000*HM(z)^alfa*K^beta+2*rand*0.5; % cobb doublas production function

PIBPC(z)=PIB(z)/size(FXP,1); % GDP per capita


resultado (z,1)=HM(z)';

resultado (z,2)=H(z)';

resultado (z,3)=E(z)';

resultado (z,5)=PIB(z)';

resultado (z,6)=PIBPC(z)';

resultado (z,4)=S(z)';


%plot((1:z),resultado(:,1))% human capital per capita

%title(['Average human capital'])

%xlabel(['t ' ])

%plot((1:z),resultado(:,2))% total human capital

%title(['Capital humano total'])

%xlabel(['t ' ])

%plot((1:z),resultado(:,3)) % total emigrants

%title(['Number of emigrants'])

%xlabel(['t']);

%plot((1:z),resultado(:,4))% students

%title(['Estudantes'])

%xlabel(['t'])

%plot((1:z),resultado(:,5))% GDP

%title(['GDP'])

%xlabel(['t'])

%plot((1:z),resultado(:,6))% GDP per capita

%title(['GDP per capita'])

%xlabel(['t'])

End
```

5.10 Discussion

Brain drain and economic growth are topics with long tradition in economic analysis. The most recent strand of research tried precisely to evaluate some non-linear phenomena regarding brain drain and is effects in the source economies.

Our aim with this paper was to test this relation and, in particular, the beneficial brain drain hypothesis. We do it using an agent-based model based on Beine, Docquier, and Rapoport (2001) with some departures from the original theoretical version, namely the many periods instead of just two, risk aversion heterogeneity and different assumptions for population growth.

It is a macroeconomic model with overlapping generation and endogenous growth features that can be improved in several ways. With the appropriate calibration, can also be useful for empirical analysis using real data.

We find that, as in the original theoretical model, emigration probability is a crucial parameter to determine brain drain consequences. In fact, economic growth can be higher or lower depending on the specific emigration probability p. We conclude too that higher individual risk aversion is related with lower economic growth and a lower beneficial brain drain effect.

6. CONCLUDING REMARKS

Our aim with this thesis was to broaden the scope of the applications of agent-based models in Macroeconomics. There are already some examples of applications of this kind of models – and computational economics in general – to deal with macroeconomic issues. But the number is still quite small compared to the variety of articles and works published about microeconomic topics.

In fact, agent-based models have almost unique features to deal with the macroeconomic reality, considering that in recent years, particularly after the 2007/2008 financial crisis, traditional models have lost some of their credit. In particular, DSGE models (dynamic, stochastic and general equilibrium) which, even with the neo-Keynesian version of the so-called New Neoclassical Synthesis, have some assumptions that have been strongly disputed in the recent years in terms of adherence to economic reality.

Representative agents and rationality are the two most criticized characteristics of these models. Because they imply, on the one hand, a degree of homogeneity that does not exist in the real world and, on the other hand, they assume a cognitive capacity of economic agents who, based on the numerous results of psychology and neurosciences, are far beyond the real capacities of the human being. For example, dealing with uncertainty and randomness, valuing time or anticipating complex phenomena with simple heuristics.

The simplifying assumptions of DSGE models are a necessary bad to assure its analytical solution but, at the same time, are a very strict constraint. It is very difficult to abandon completely this hypothesis, without compromising the mathematical solution of the model, tough there are some models with less simplifying approaches.

Agent-based models are a tool to introduce heterogeneity in the models, to artificially create a more complex economy and to abdicate of the rationality hypothesis. These models allow us to introduce also a series of random variables in order to give a more realistic trace to the behavior of the system we are analyzing. Economies are complex systems, like many other systems in the nature, and the computational methods are a valuable option to deal with this complexity.

In this paper, we present three different applications of agent-based models to macroeconomics. These are three very different models that illustrate the possible paths that agent models in macroeconomics can follow in future.

In Chapter 3, an overlapping generation-endogenous growth model to assess how individual agent satisfaction interferes with human capital (education) and long-term growth. More specifically, we analyzed how different weights for absolute and relative income (compared to others) in satisfaction affect these results in order to evaluate the Easterlin paradox, which suggests that relative returns may be more relevant to happiness than absolute income. Our simulations showed that the scenario in which both components have equal weight is the best performer. On the contrary, growth is lower when satisfaction depends only on relative income and, when it only results from absolute income, it is similar to the baseline scenario.

The chapter 4 model is intended to simulate contagion of sovereign defaults based on a model of game theory by Jean Tirole. We found that higher risk aversion levels were associated with fewer defaults, that the contagion process can be very fast and that, among other things, monetary policy has concrete advantages in preventing sovereign debt defaults but can create moral hazard problems.

Finally, in chapter 5, we present a model to test the brain drain phenomena and, in particular, the beneficial brain drain hypothesis to the source economy due to the accumulation of human capital. Emigration probability is the crucial variable, measuring the part of potential emigrants who can actually leave the country. We concluded that this probability has a non-linear relationship with economic growth and that higher levels of risk aversion of agents affect economic performance and the beneficial brain drain.